Proving Information Flow Security for Concurrent Programs

Marco Eilers
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(Automatic) Program Verification

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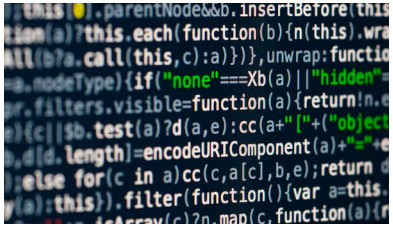

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(e.g., sort algorithm)

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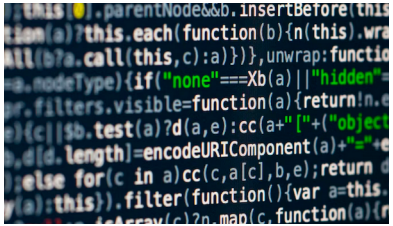

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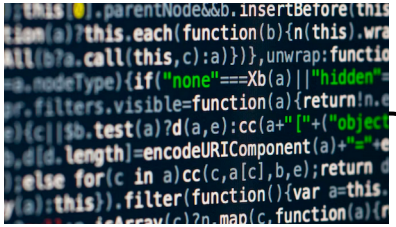

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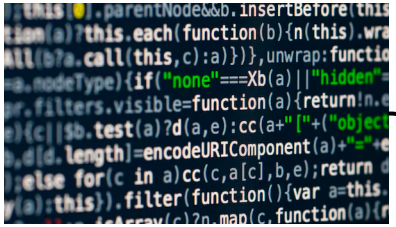

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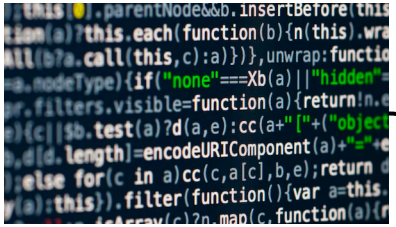

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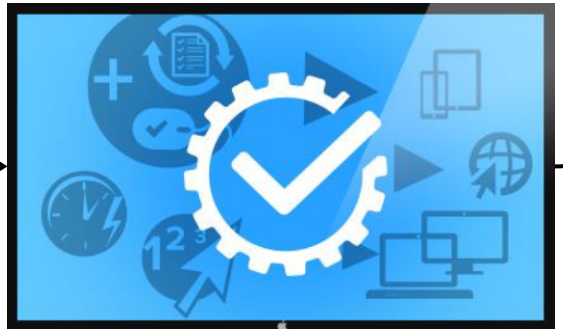

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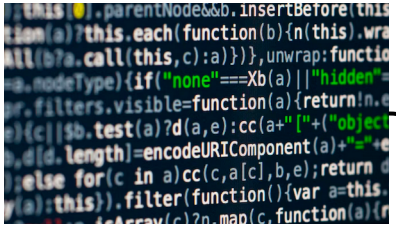

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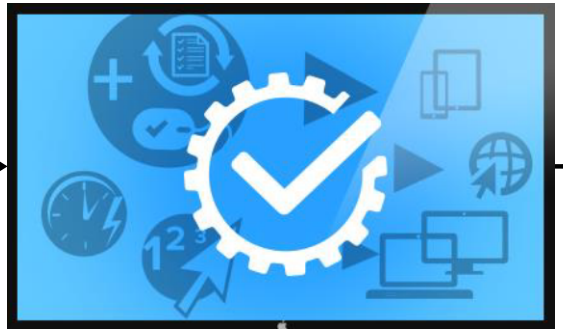

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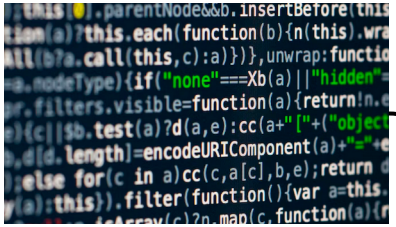

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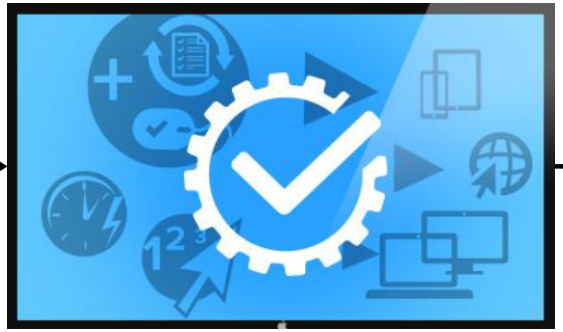

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Program **might** violate specification
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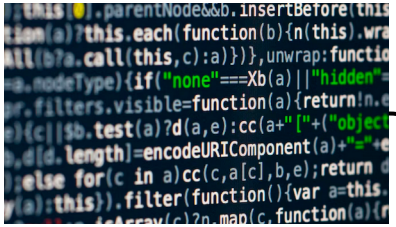

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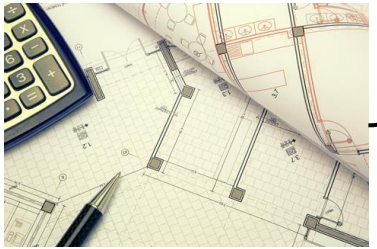

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(e.g., the output is sorted)

Hints (e.g., loop invariants)

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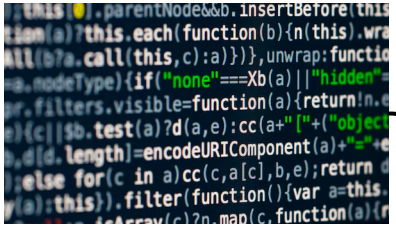

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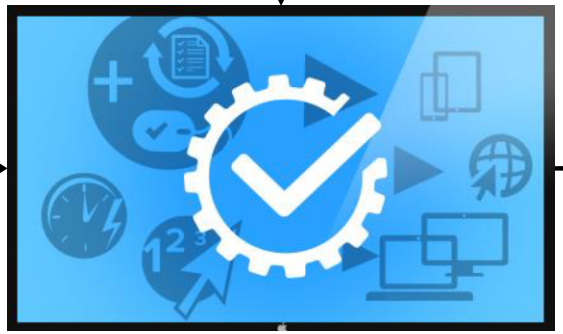

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Program **might** violate specification
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Can include **security** properties
(e.g., information flow security)

Secure Information Flow: Value Channel

```
def compute(h: int, l: int):
  if h > 0:
    res = 1
  else:
    res = 2
  return res
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Secure Information Flow: Value Channel

high-sensitivity (secret)

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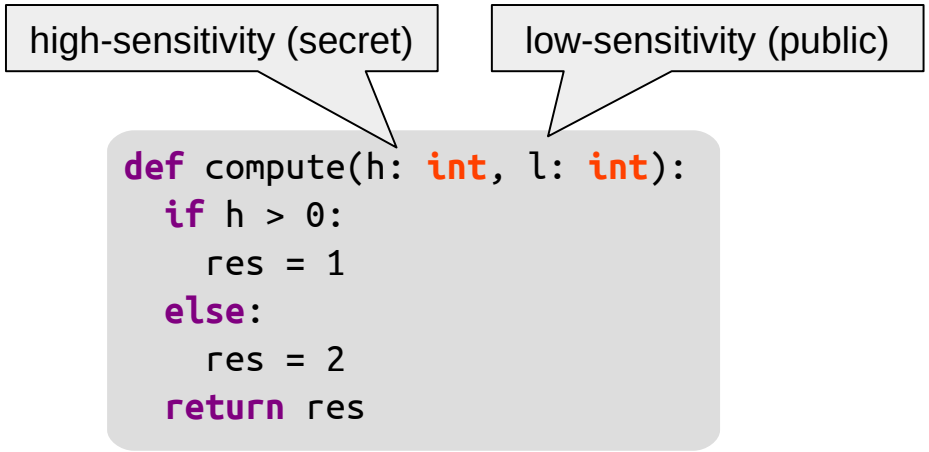

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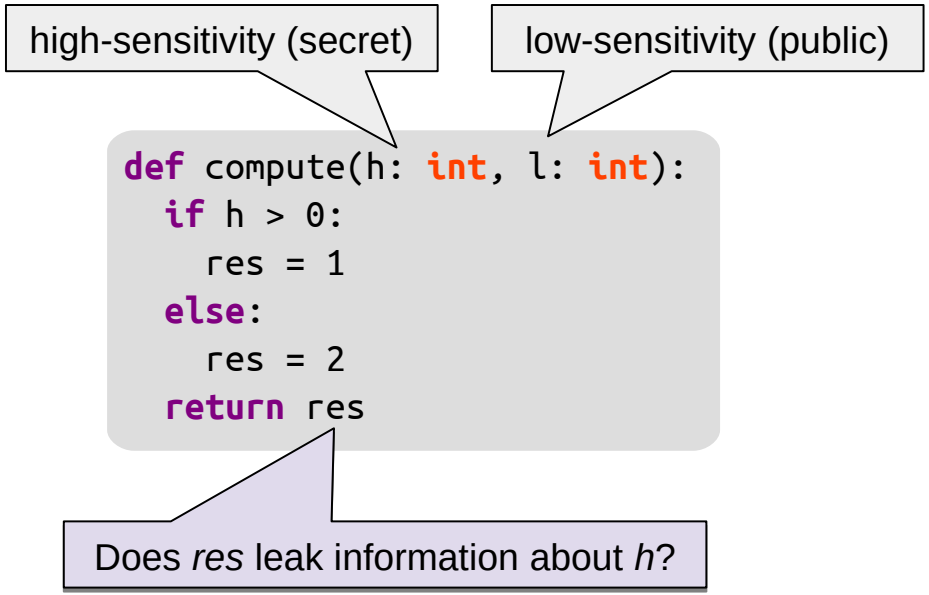

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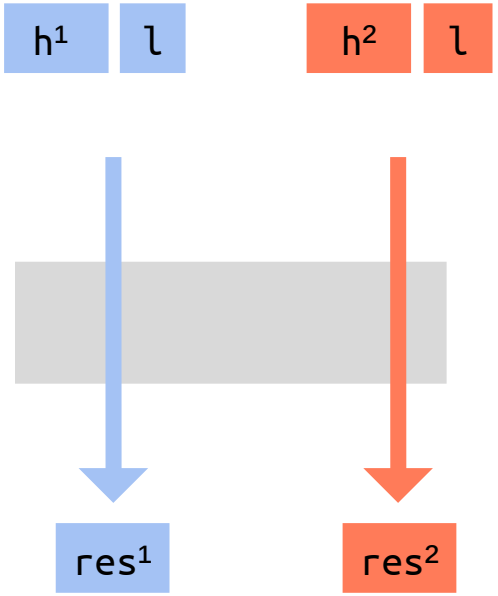
Does *res* leak information about *h*?



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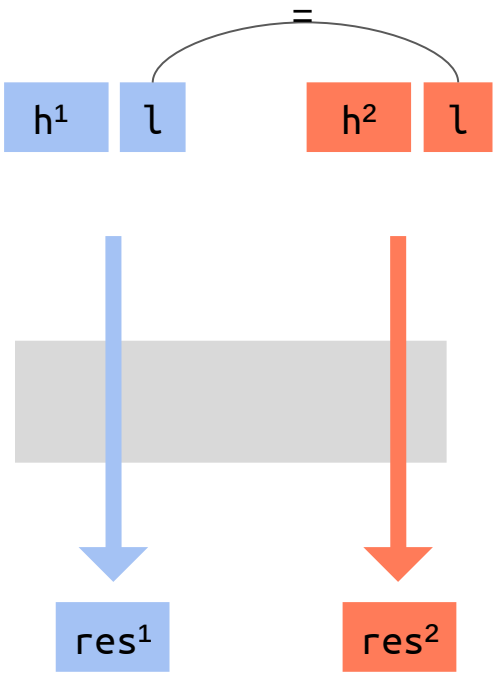

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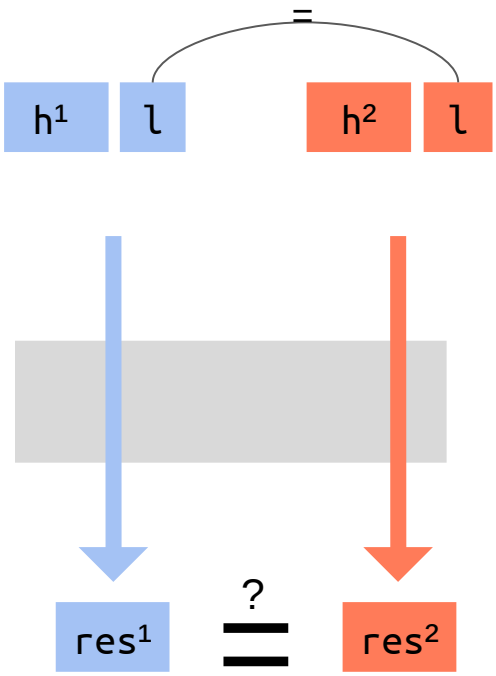

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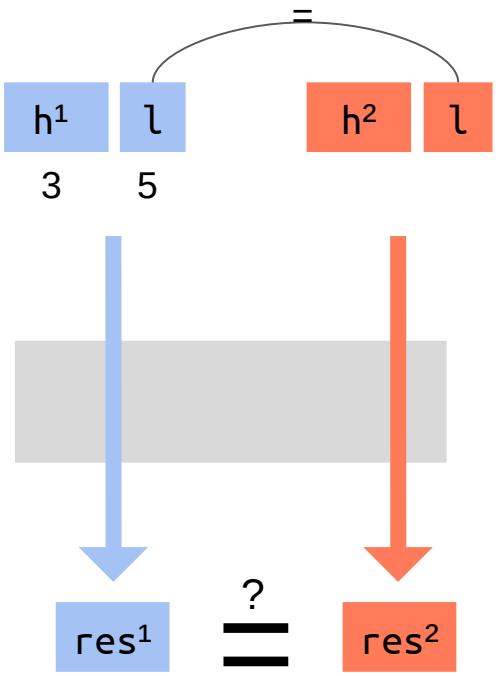

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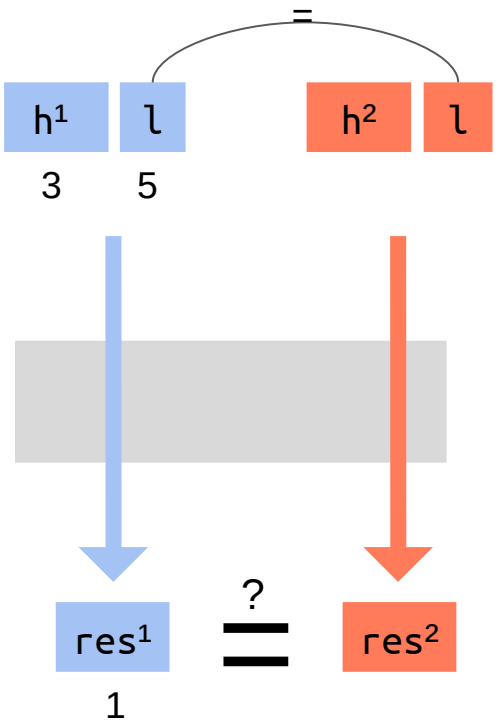

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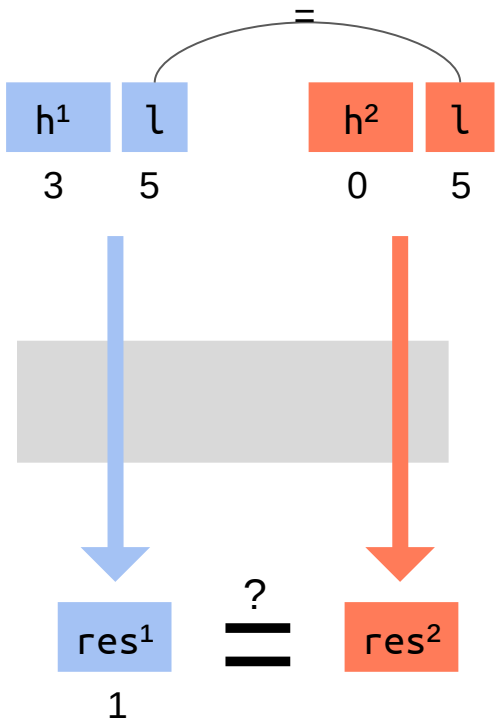

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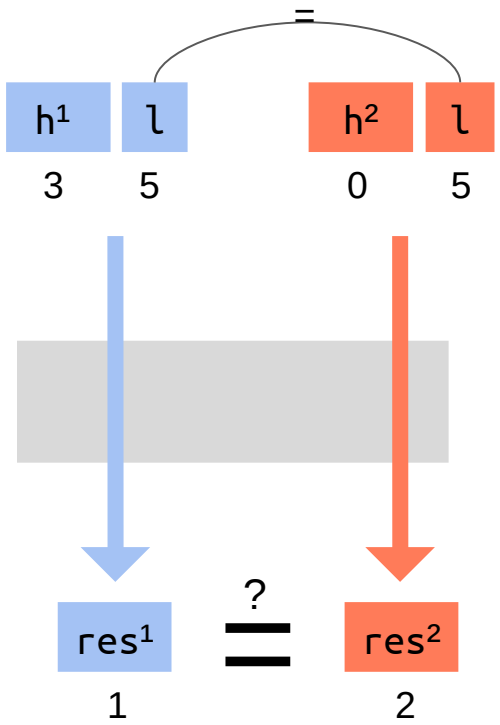

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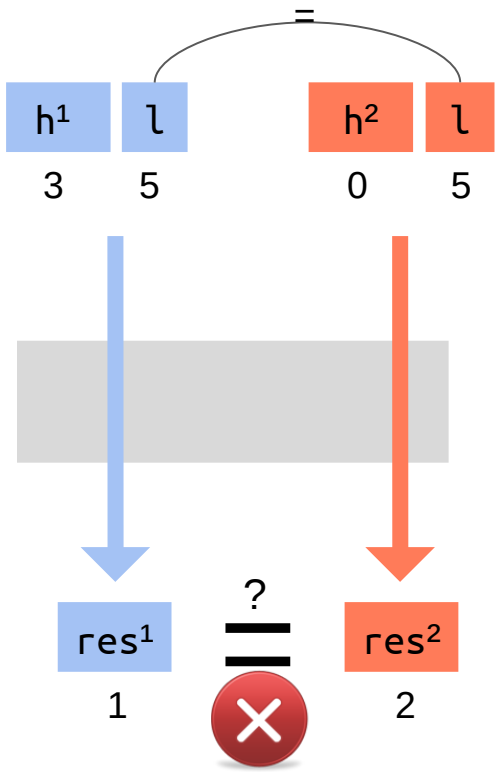

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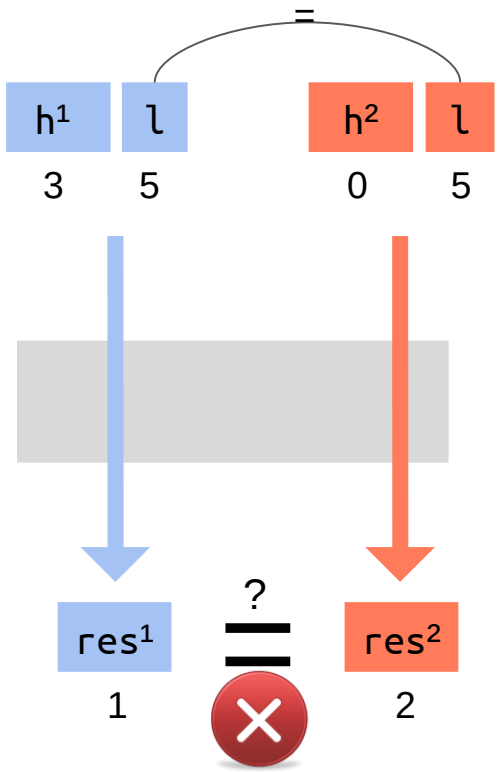

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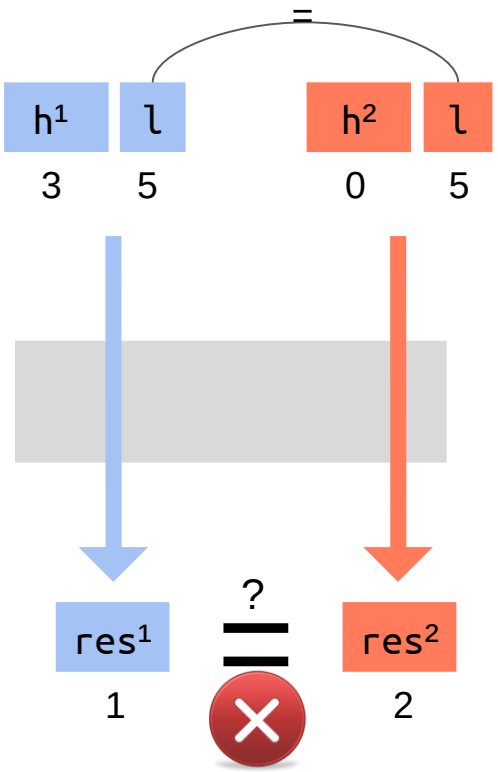

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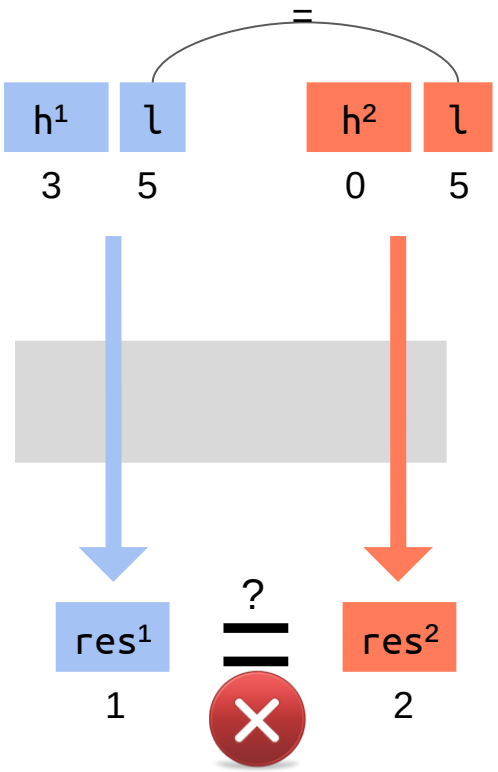

Secure Information Flow: Timing Channel

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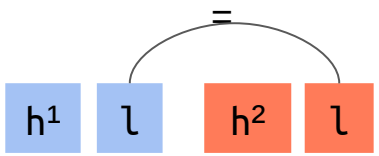

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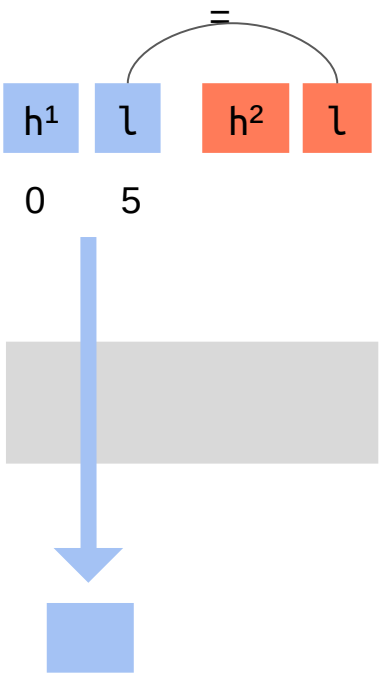

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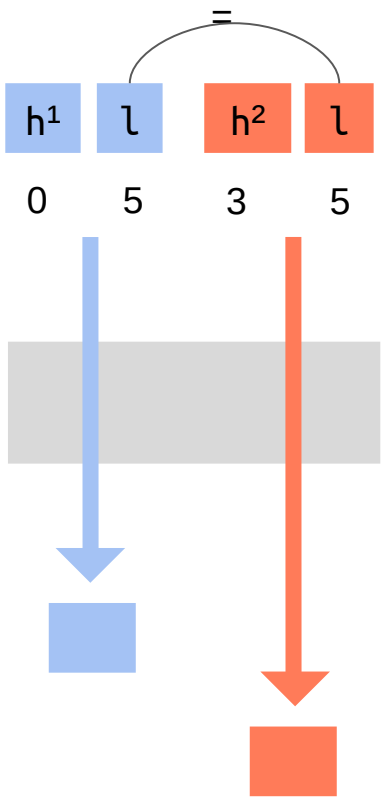

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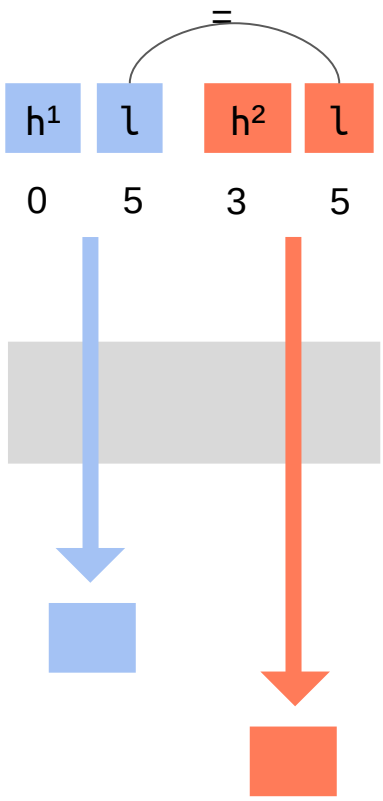

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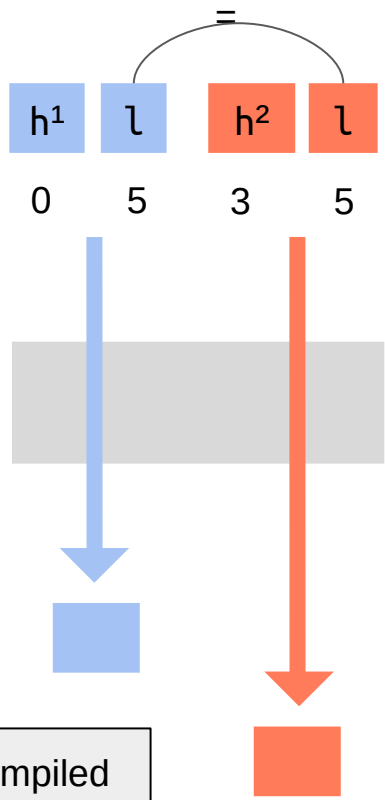


```
          section   .text
_start:   mov       rax, 1
          mov       rdi, 1
          mov       rsi, message
          syscall
          ...
```

The execution time of the compiled program typically depends on values

Secure Information Flow

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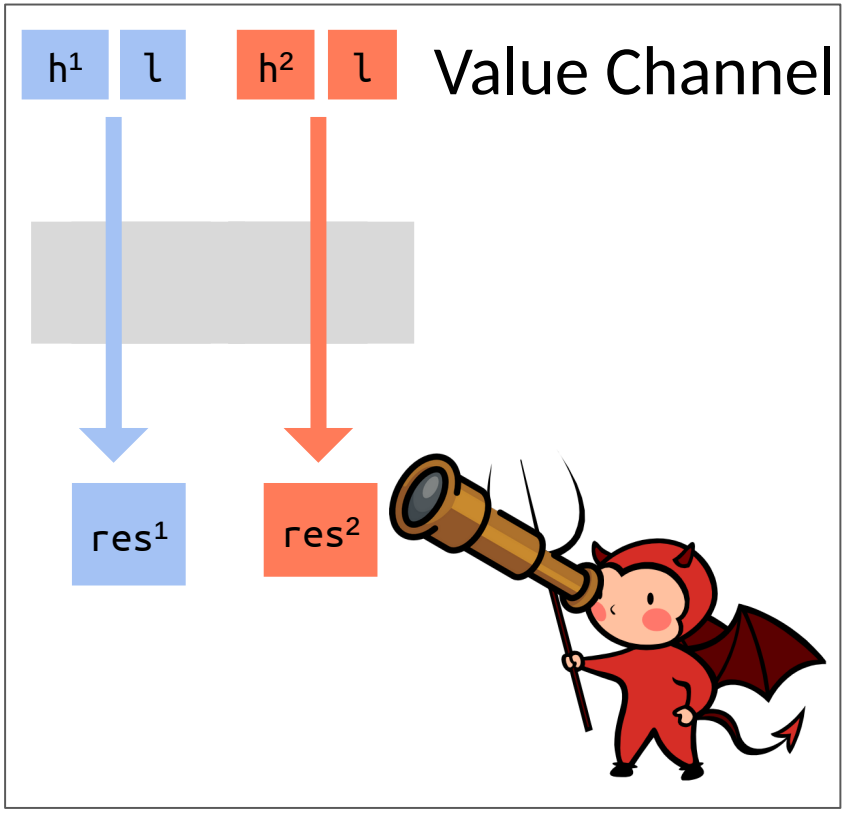

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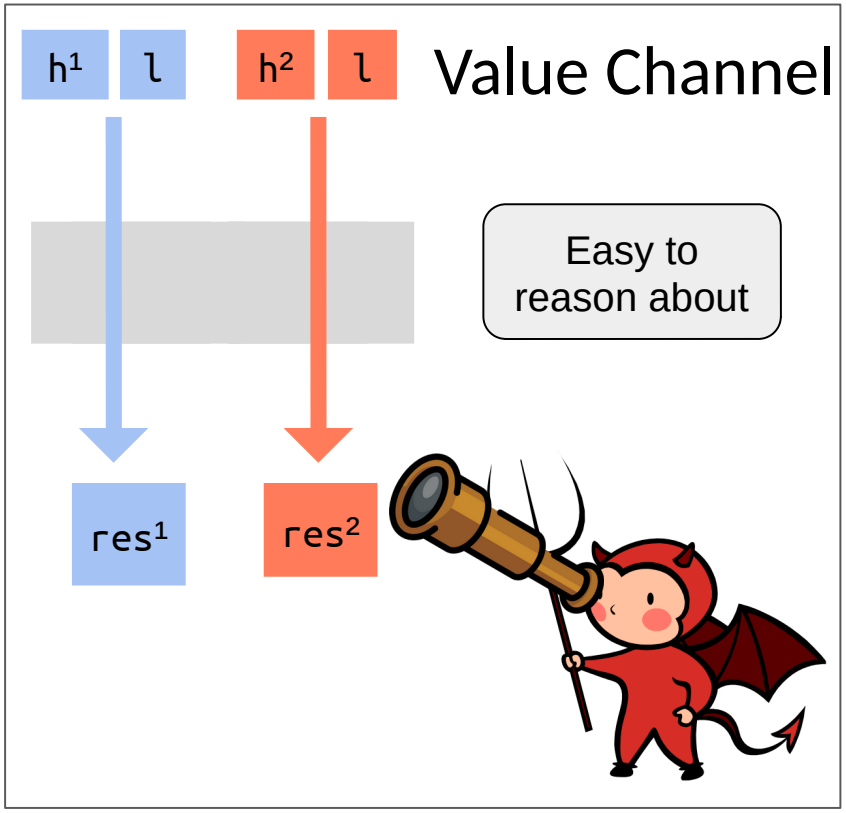

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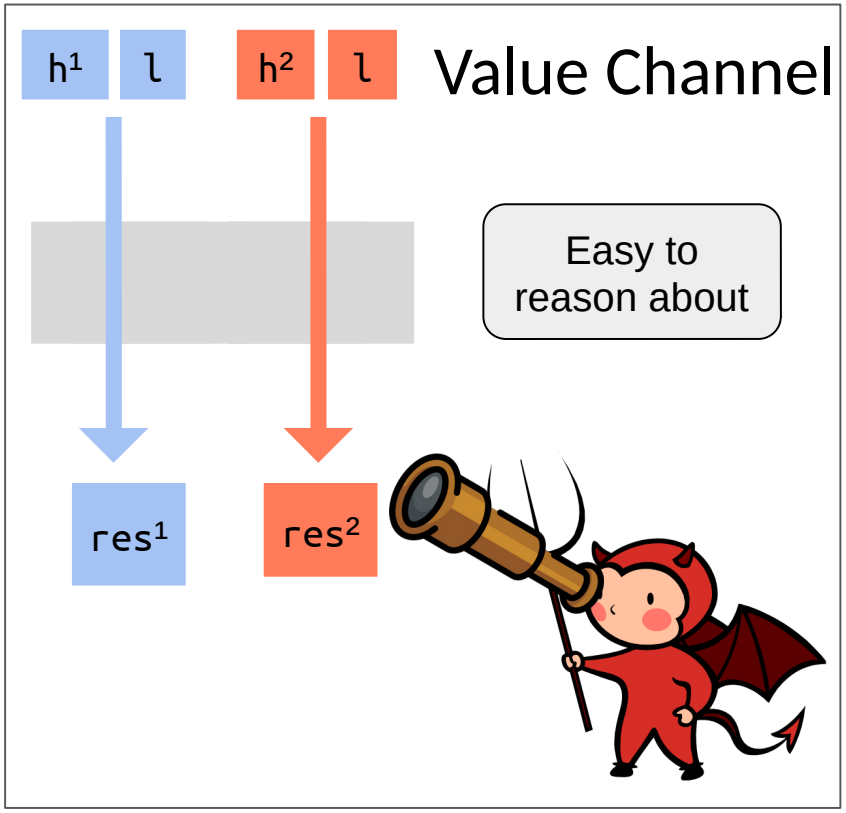

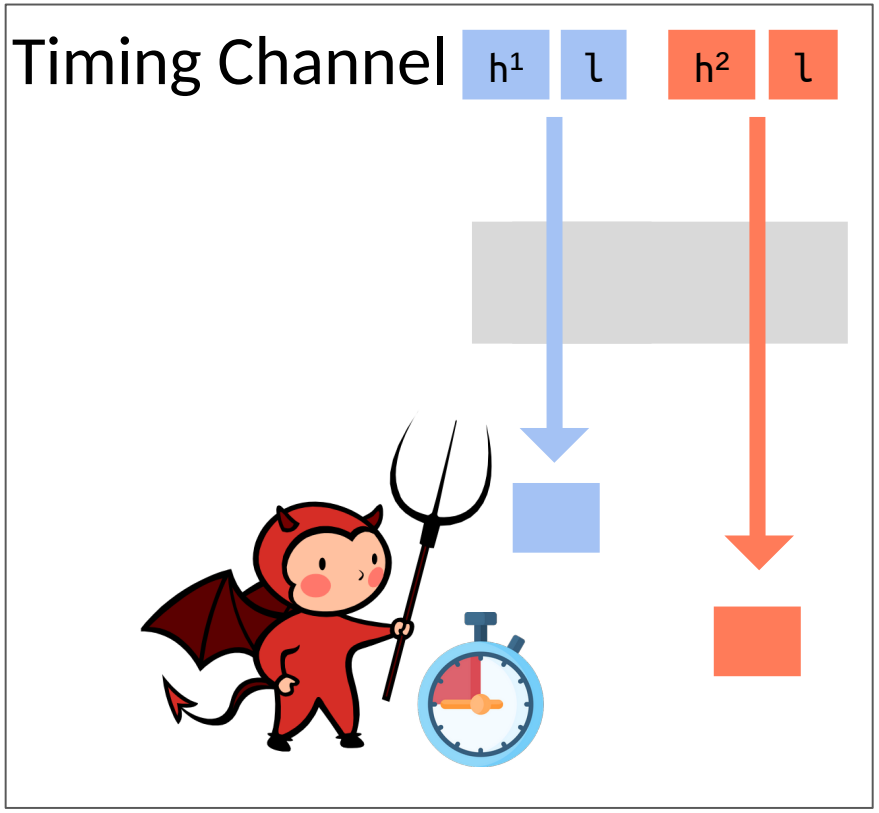

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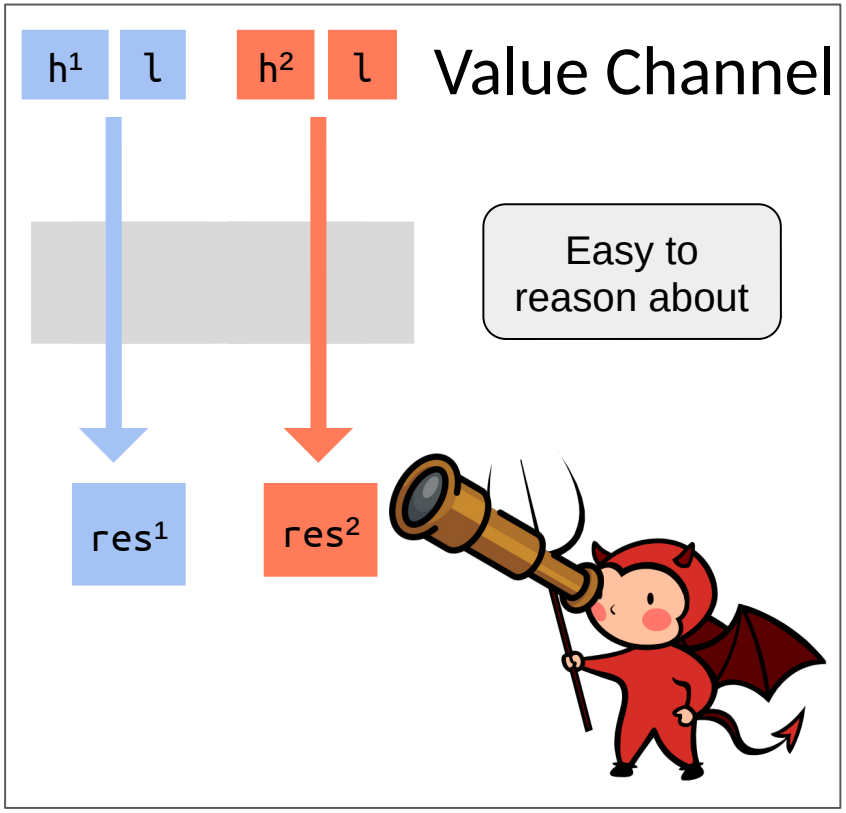
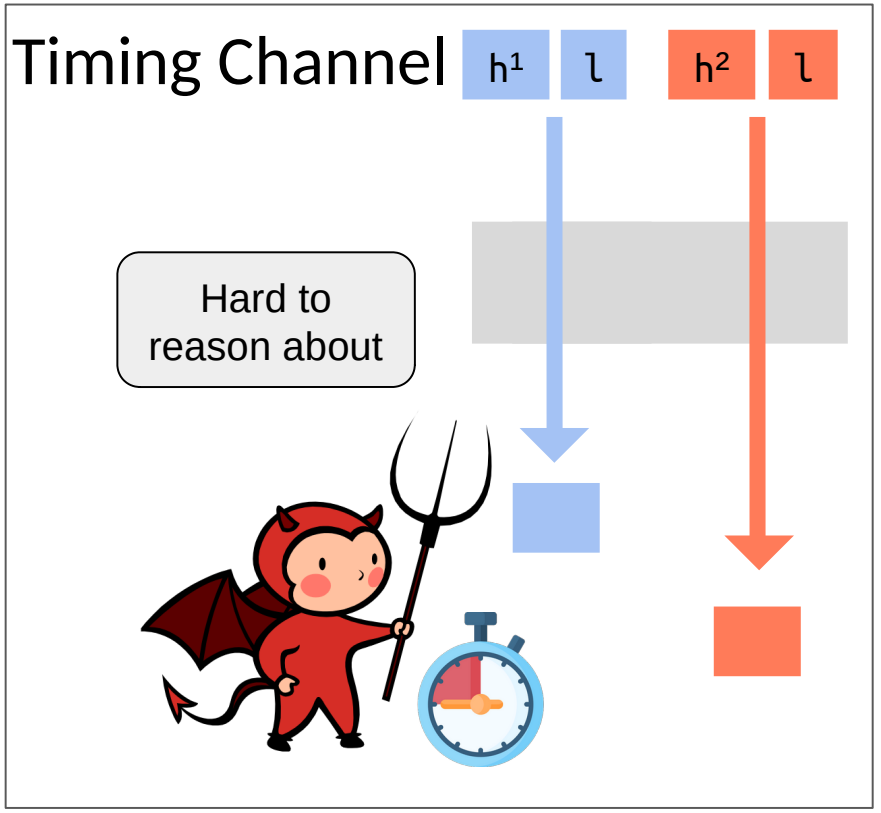
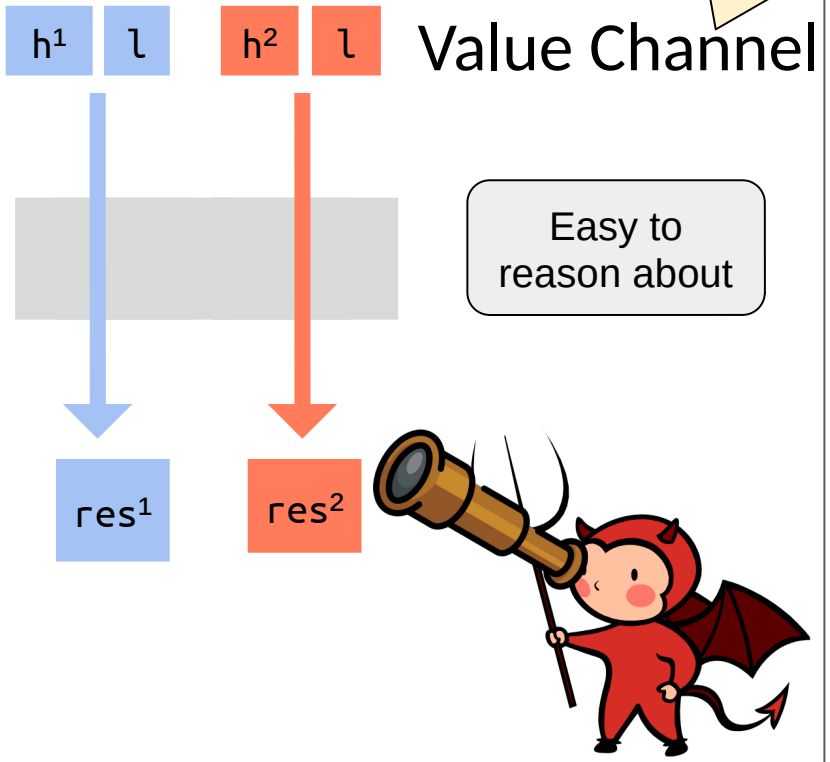




Secure Information Flow

This talk

Value Channel



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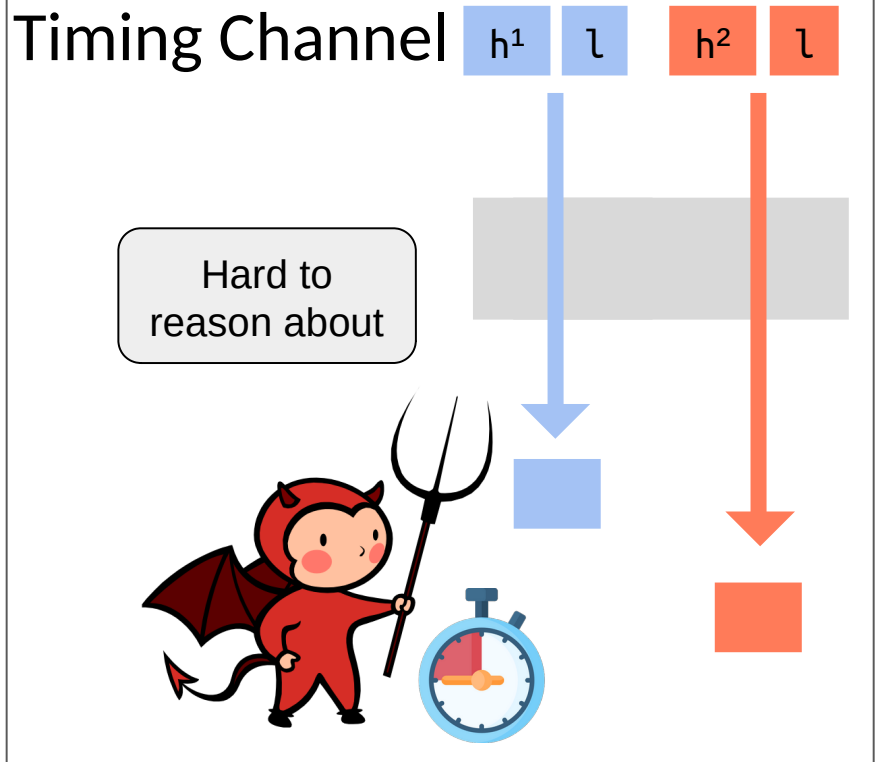

Attacker:
Observes **final results**,
not intermediate state or **timing**

Shared-Memory Concurrency Ruins Everything

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```
while i < h:          ||  while j < 100:
  i += 1              ||    j += 1
shared = 6            ||  shared = 7

        return shared
```

Shared-Memory Concurrency Ruins Everything

Secret-dependent execution time

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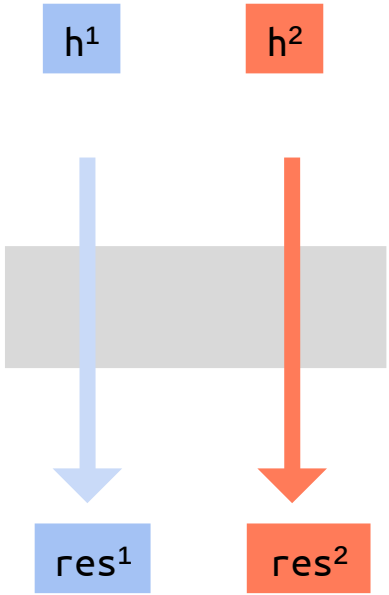

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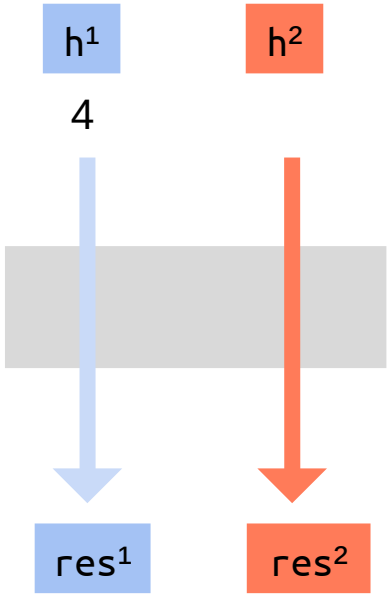

Shared-Memory Concurrency Ruins Everything

Secret-dependent execution time

Secret-**in**dependent execution time

```
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  i += 1              ||    j += 1
shared = 6            ||  shared = 7

return shared
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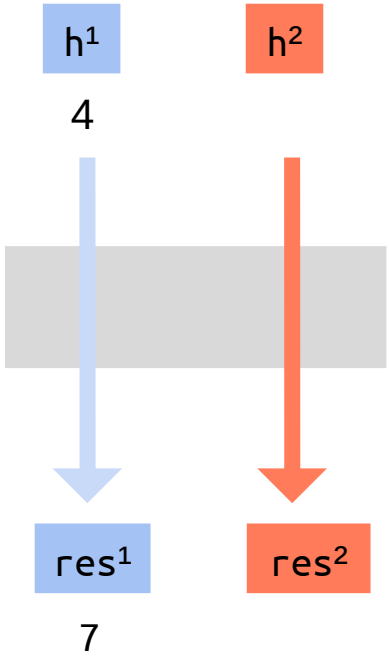

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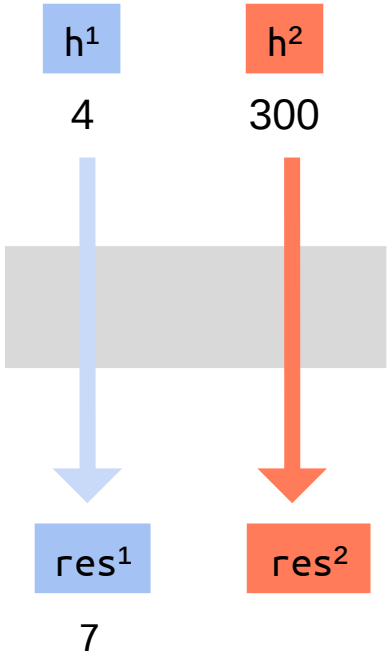

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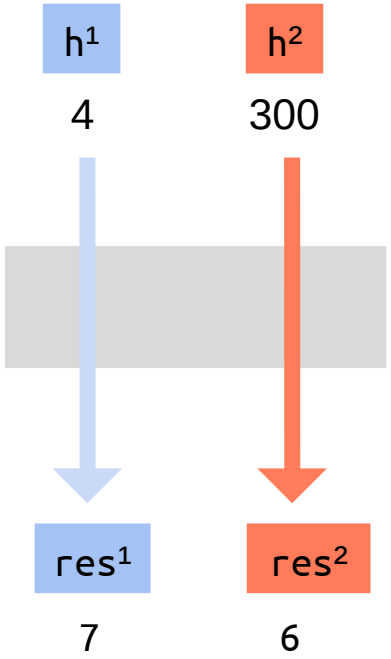

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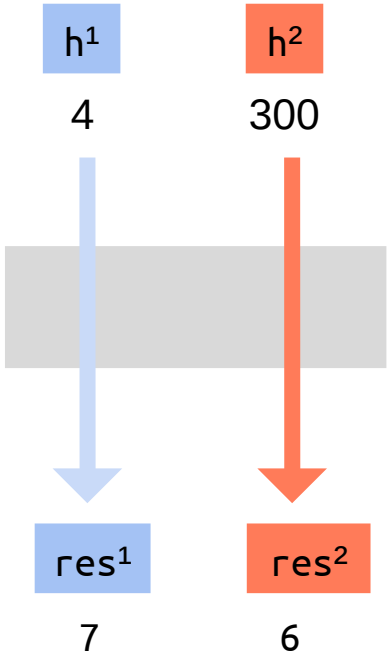

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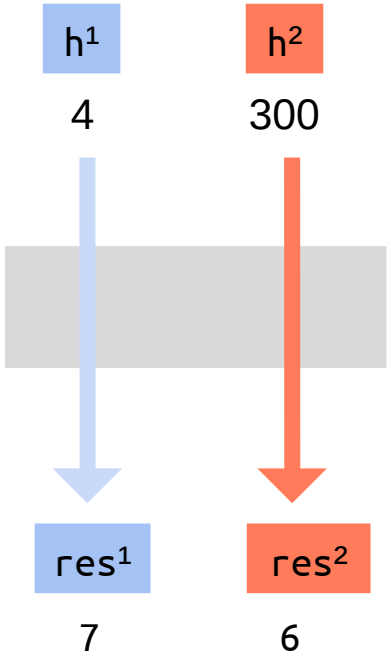


Reasoning about Value Channel

Easy

Reasoning about Value Channel

Easy

Reasoning about Value Channel + Concurrency

Easy

Reasoning about Value Channel + Concurrency

Reasoning about Timing Channel

Easy

Reasoning about Value Channel + Concurrency

Reasoning about Timing Channel

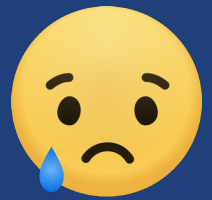
Hard

Easy

Reasoning about Value Channel + Concurrency

Reasoning about Timing Channel

Hard



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```

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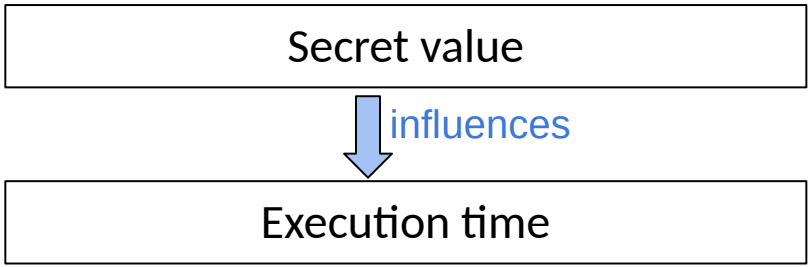
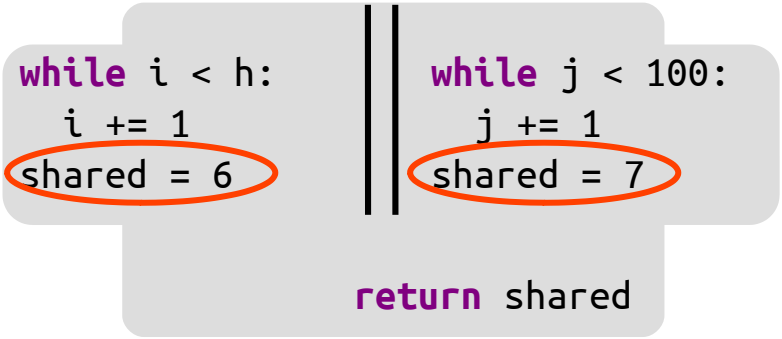
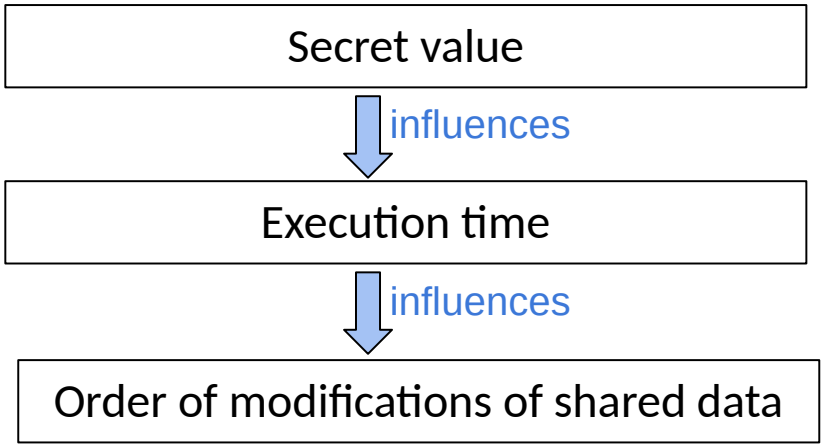


Shared-Memory Concurrency Ruins Everything





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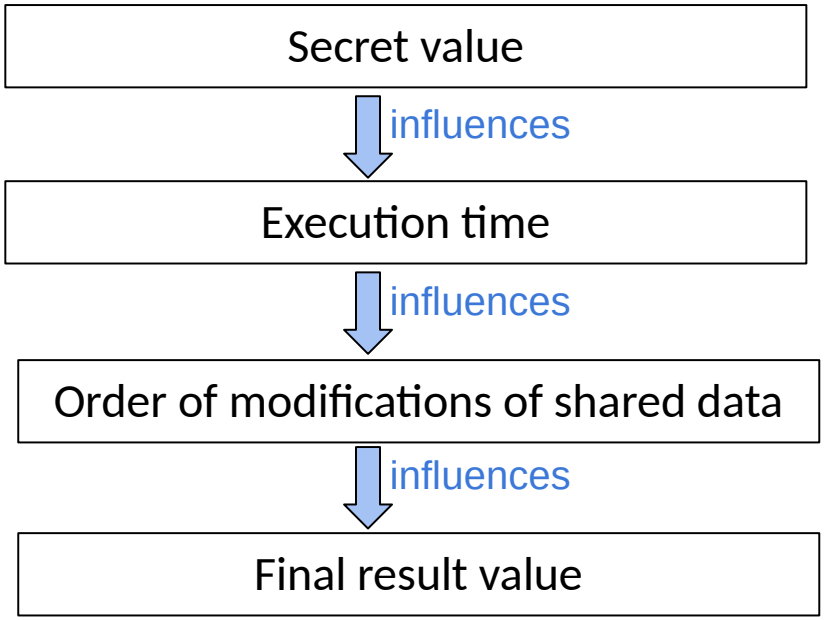

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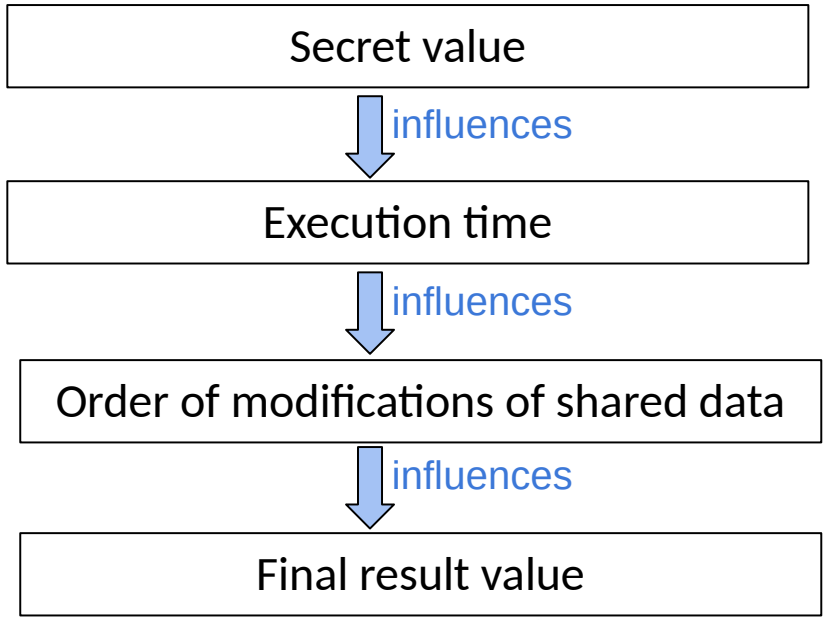
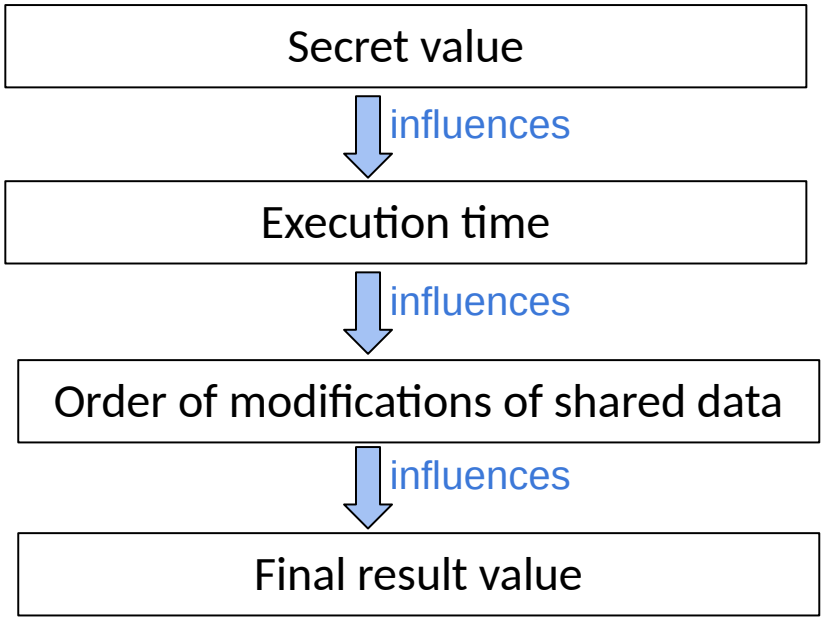


Existing (Modular) Solutions

```
while i < h:          ||  while j < 100:
  i += 1              ||    j += 1
shared = 6            ||  shared = 7

                  return shared
```



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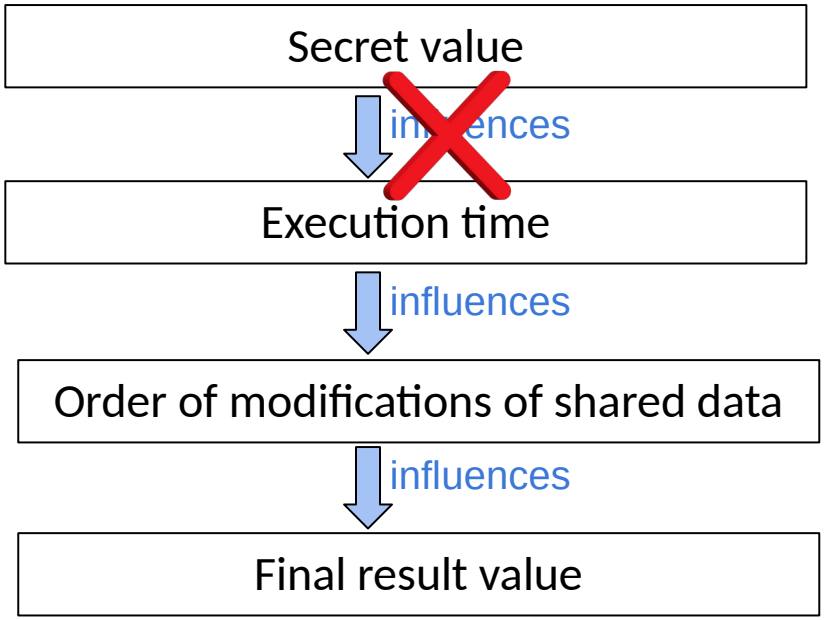

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while i < h:          ||  while j < 100:
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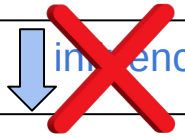
```

Secret value



Execution time

influences

Order of modifications of shared data

influences

Final result value

Existing (Modular) Solutions

Insecure

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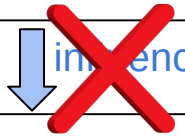
shared = 6            ||  shared = 7

return shared
```

Secret value



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```
while i < h:          ||   while j < 100:
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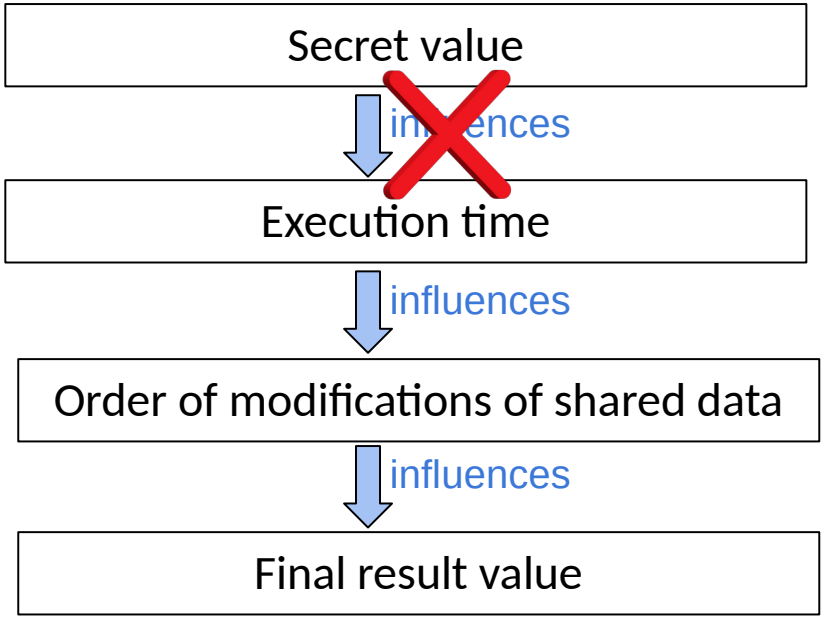
shared = 6            ||   shared = 7

                      return shared
```

```
                      shared = l
while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
atomic:               ||   atomic:
  shared += 6         ||     shared += 7

                      return shared
```



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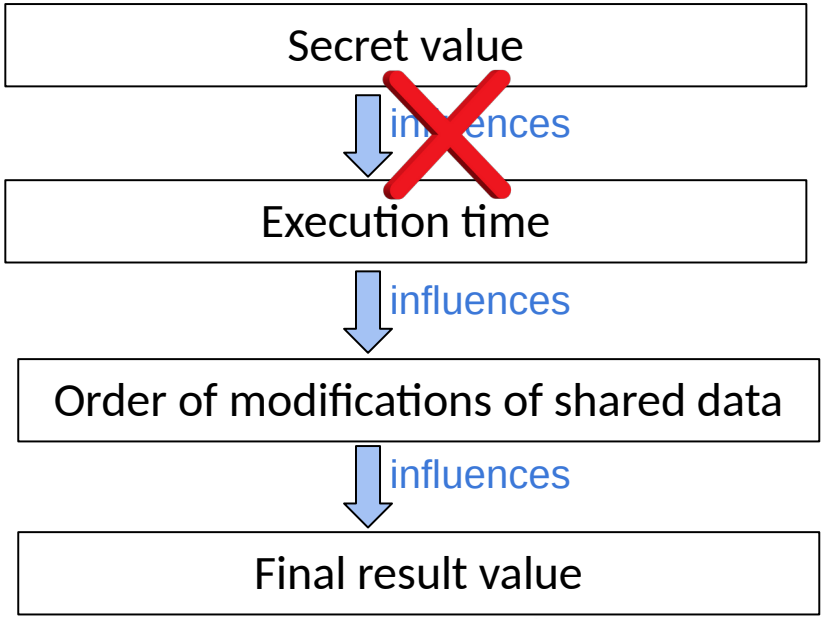

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```
                      shared = l
while i < h:          ||   while j < 100:
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```

Secret value

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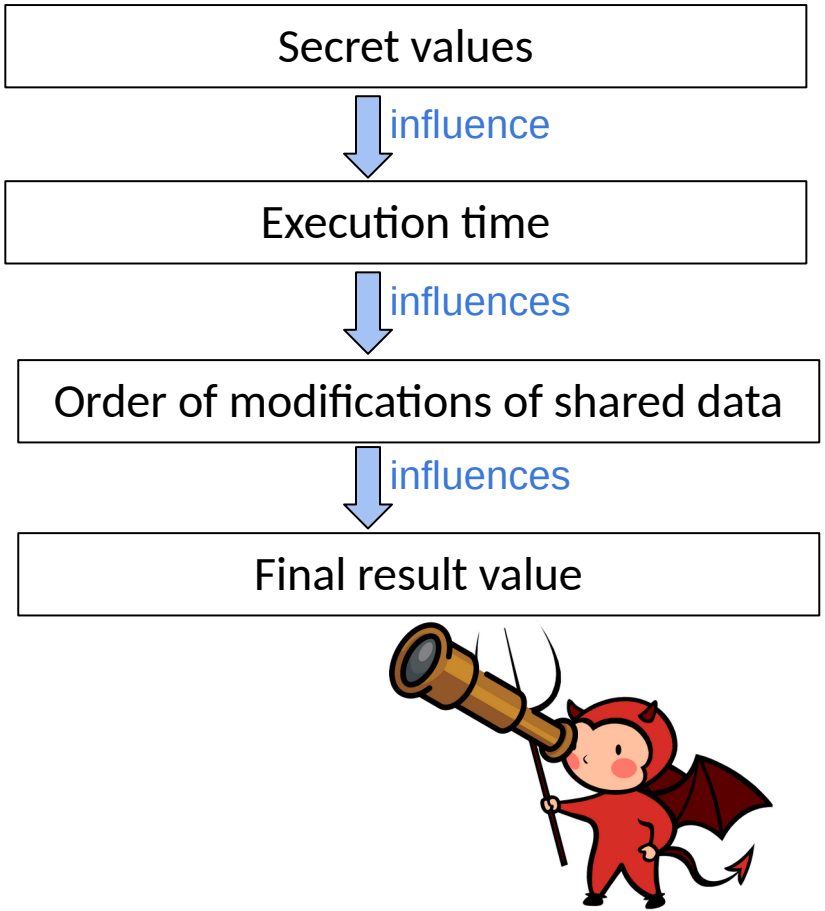
Problem Statement

Reason about **values** in concurrent programs
without reasoning about **timing**
and without considering all **interleavings**

Key Idea

Order does not influence result if modifications **commute**

Our Solution: Commutativity



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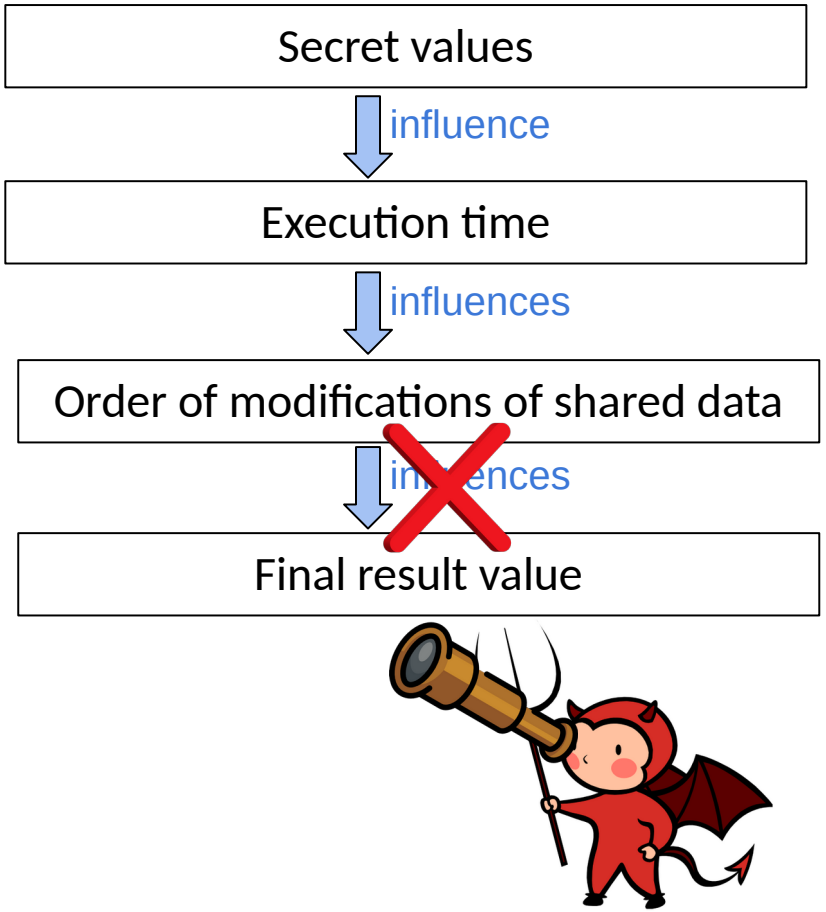

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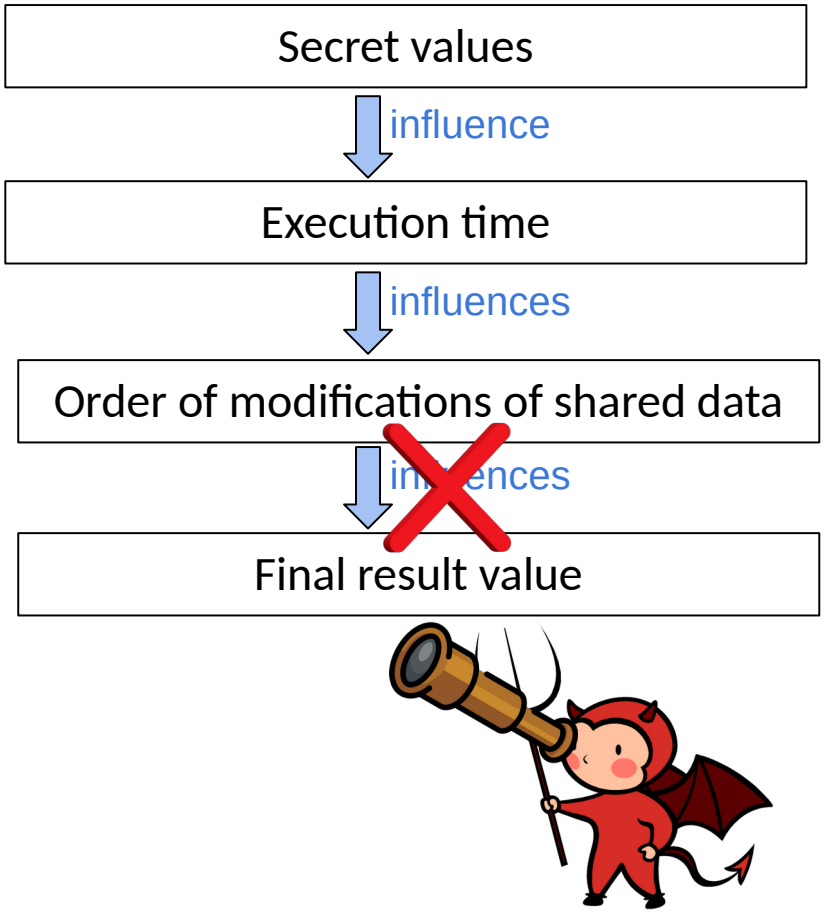

Our Solution: Commutativity

Secure

```
                    shared = l
while i < h:                    while j < 100:
  i += 1                          j += 1
atomic:                         atomic:
  shared += 6                     shared += 7
                    return shared
```

Secret values

influence

Execution time

influences

Order of modifications of shared data

influences

Final result value

Our Solution: Commutativity

Secure

```
shared = l
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  i += 1              ||     j += 1
atomic:               ||   atomic:
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Secret values

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Secret values

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Secret values

influence

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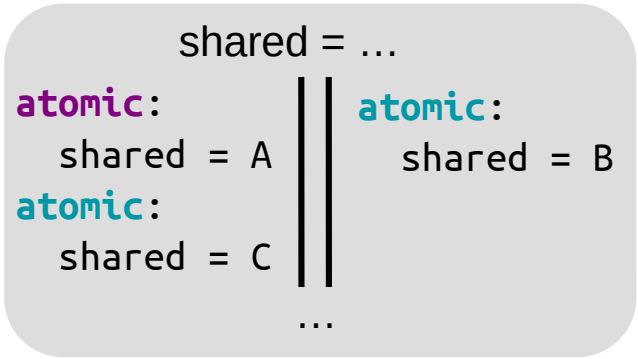
Final result value

Basic Solution



Basic Solution

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        shared = …
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atomic:          ||
  shared = C     ||
                 ||
                 …
```

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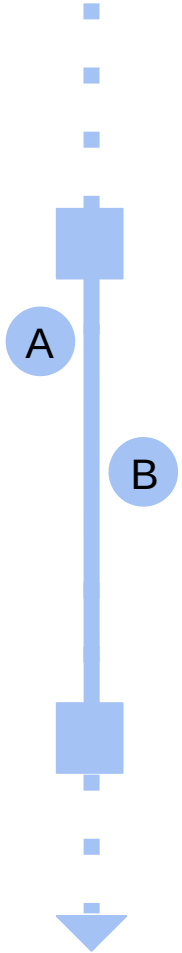

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                         ||
                         …
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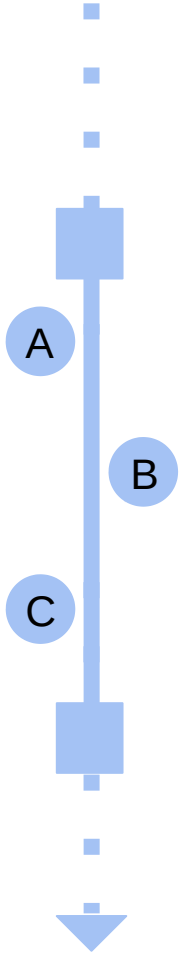

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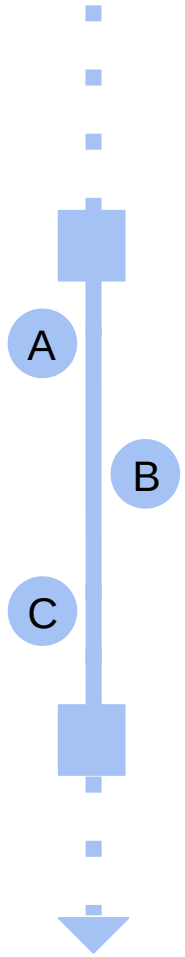

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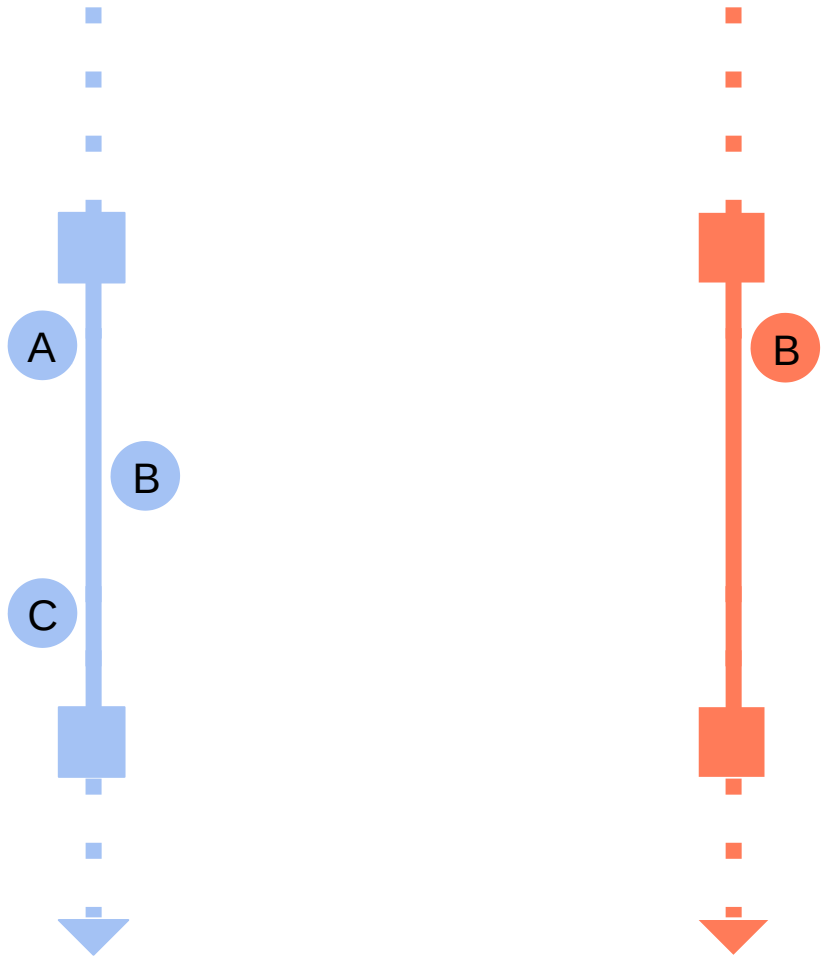

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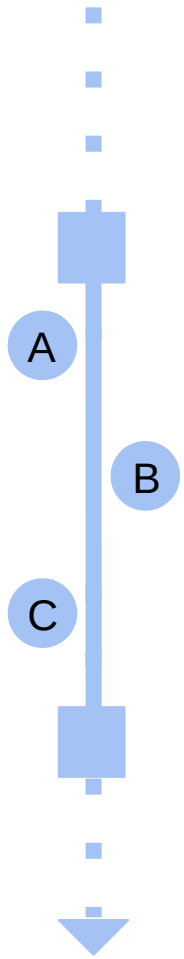

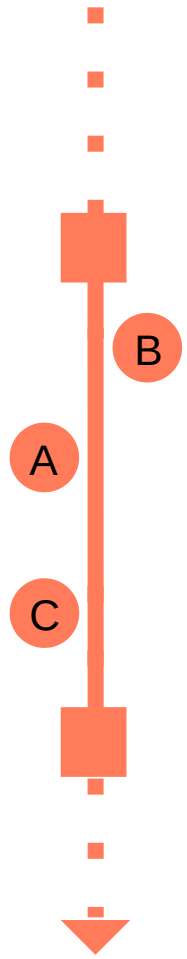

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                         ||
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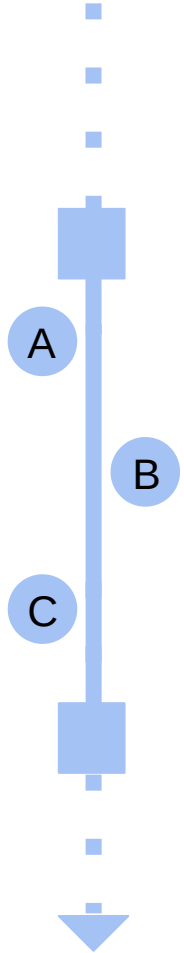

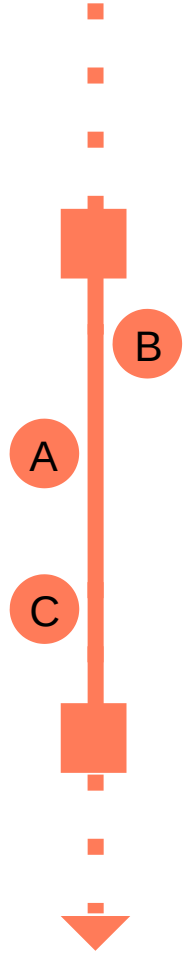

If

Basic Solution

```
shared = …
atomic:                  ||  atomic:
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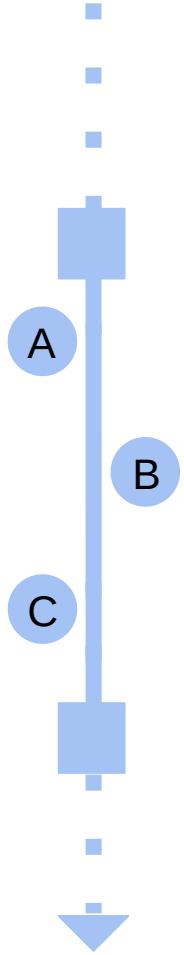
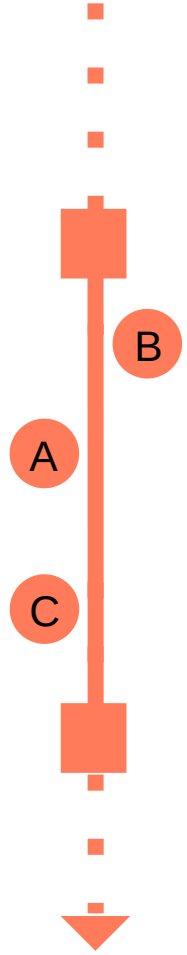
atomic:                  ||
  shared = C             ||
                         ||
                         …
```



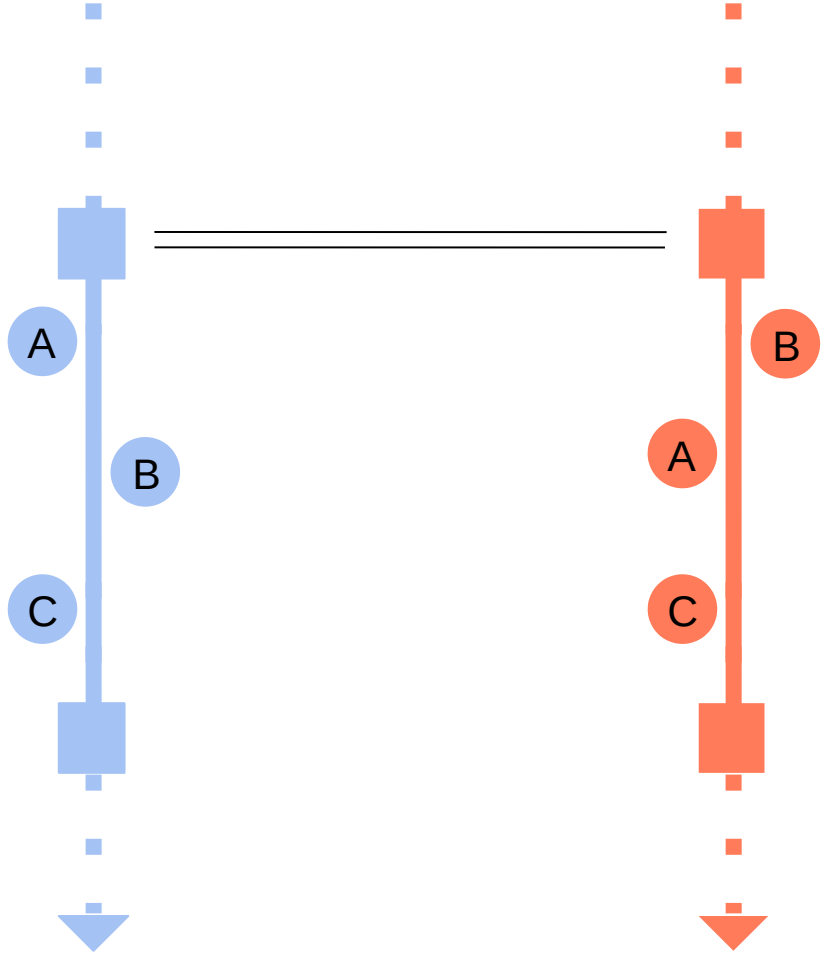


If

(1) *shared* has the same initial value in both executions

Basic Solution

```
        shared = …
atomic:          ||  atomic:
  shared = A     ||    shared = B
atomic:          ||
  shared = C     ||
                 ||
                …
```



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(1) *shared* has the same initial value in both executions

Basic Solution

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atomic:                  ||
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                         ||
                         …
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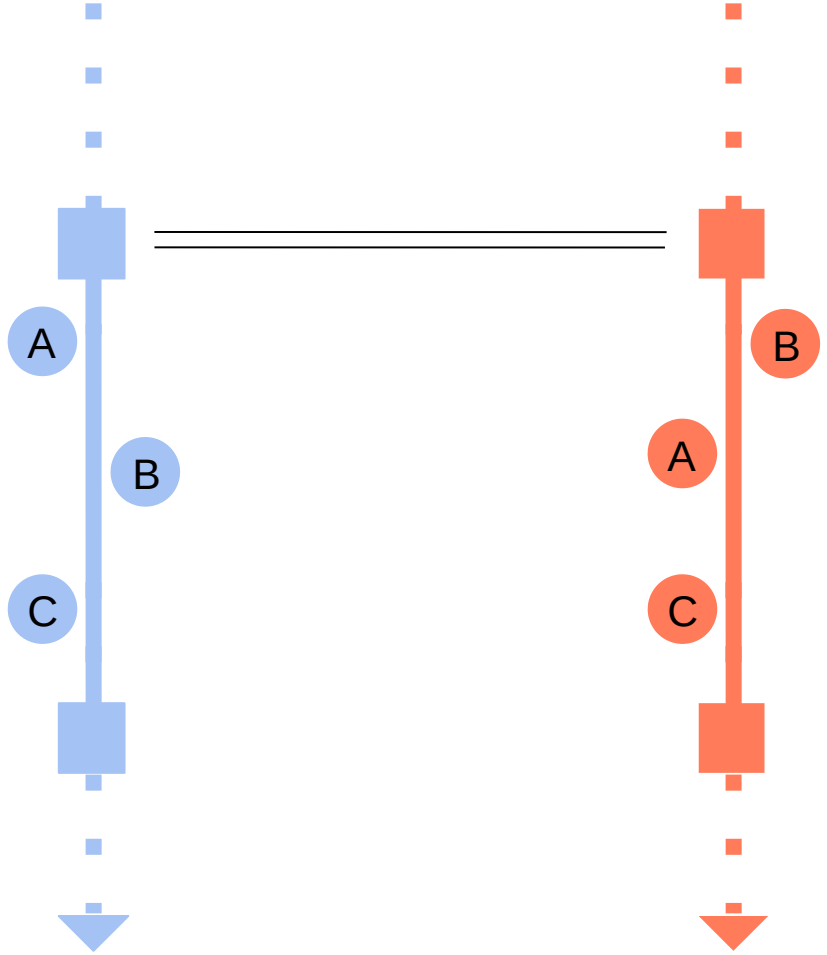

If

(1) *shared* has the same initial value in both executions

(2) the two executions perform the “same” modifications

Basic Solution

```
shared = …
atomic:                  ||  atomic:
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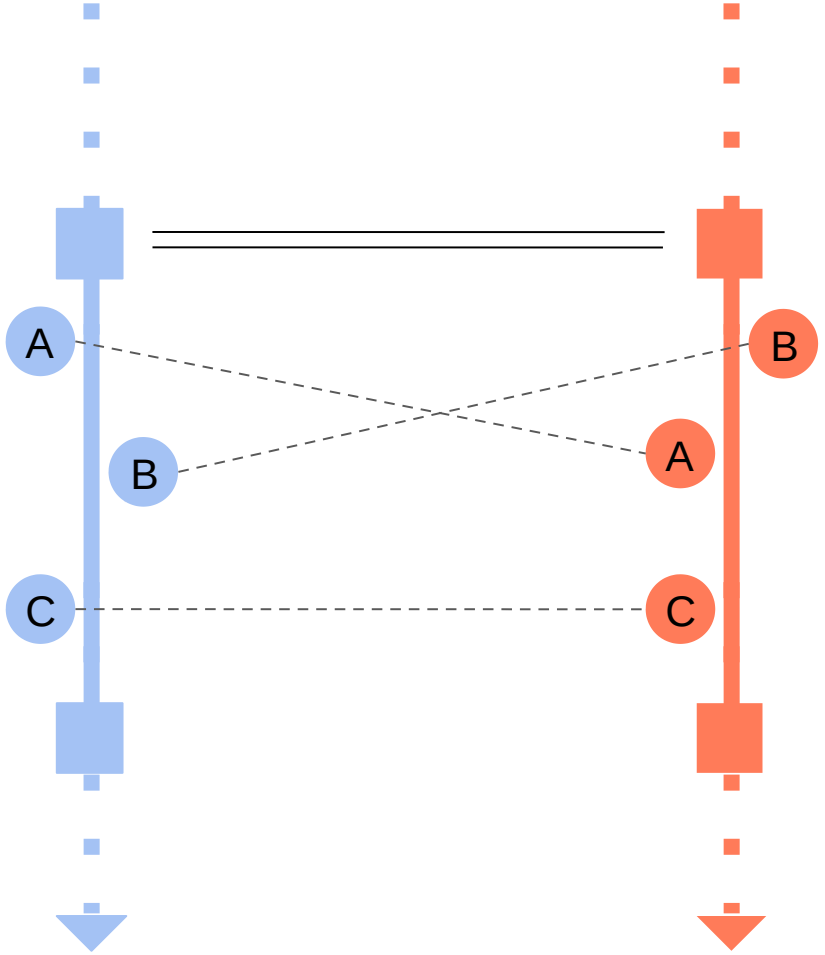
atomic:                  ||
  shared = C             ||
                         ||
                         …
```



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Basic Solution

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                         ||
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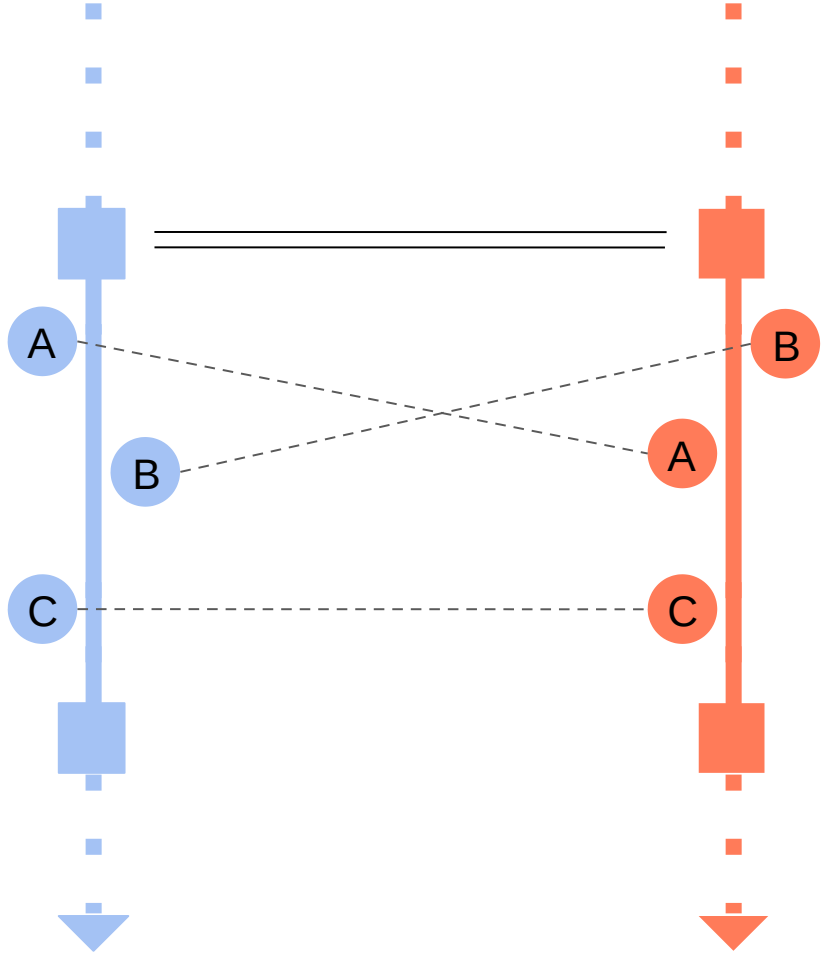

If

(1) *shared* has the same initial value in both executions

(2) the two executions perform the “same” modifications

(3) the modifications commute

Basic Solution

```
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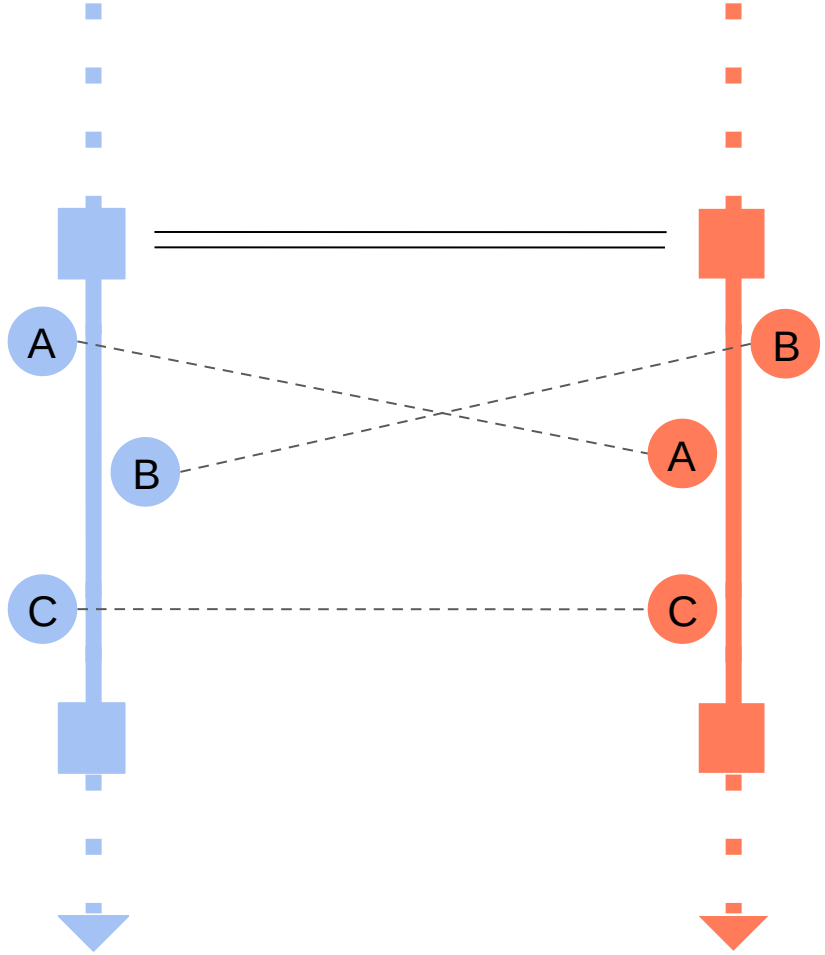
atomic:          ||
  shared = C     ||
                 ||
                …
```



If

(1) *shared* has the same initial value in both executions

(2) the two executions perform the “same” modifications

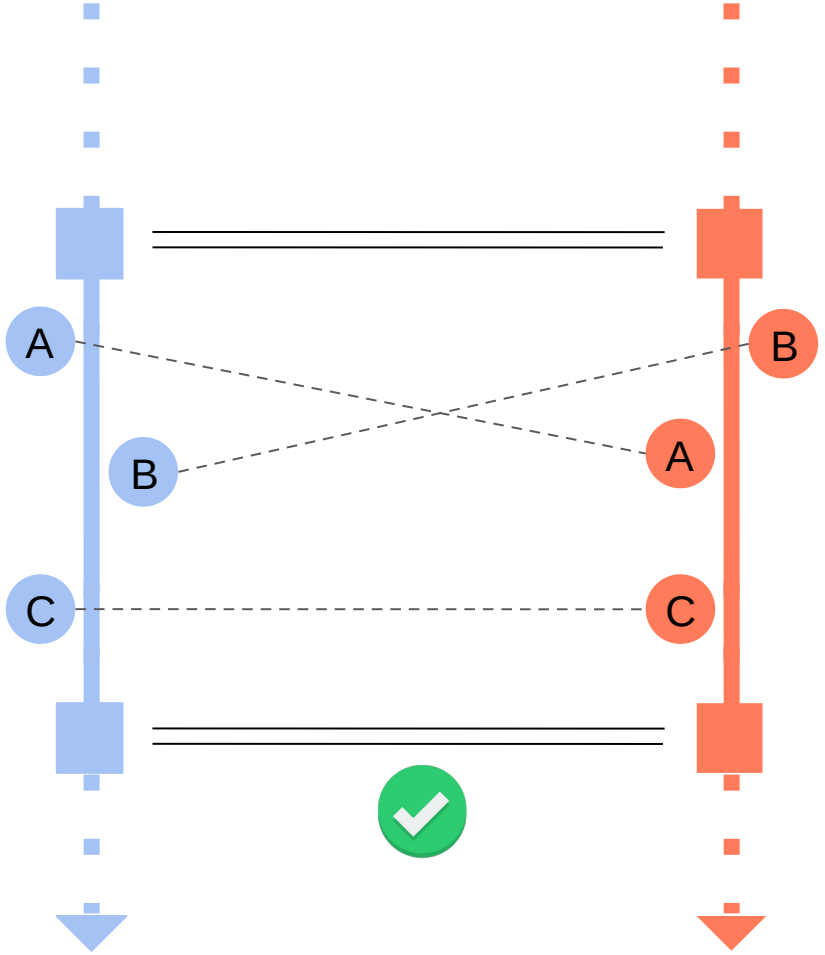
(3) the modifications commute

then *shared* has the same final value in both executions

Basic Solution

shared = …

```
atomic:          ||  atomic:
  shared = A     ||    shared = B
atomic:          ||
  shared = C     ||
                 ||
                 …
```



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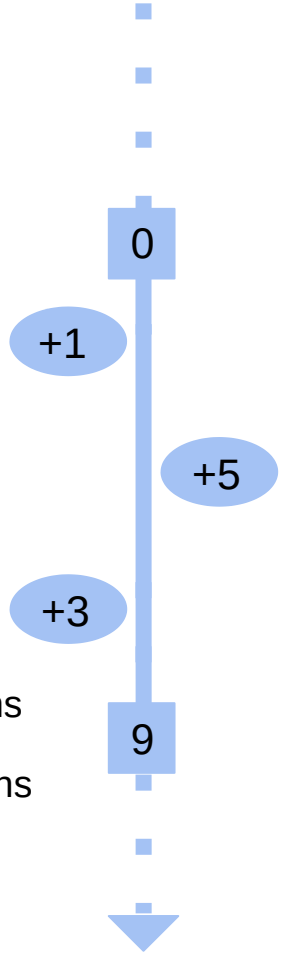

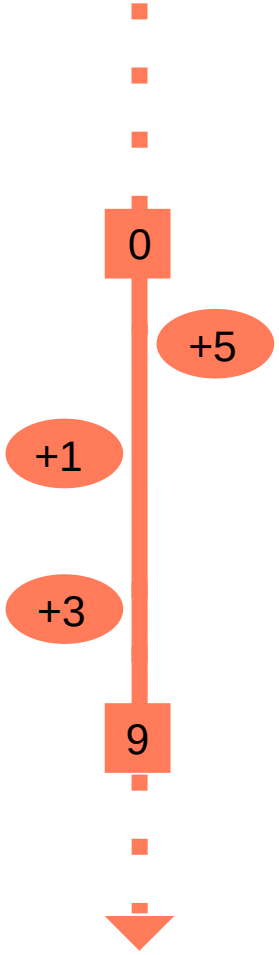

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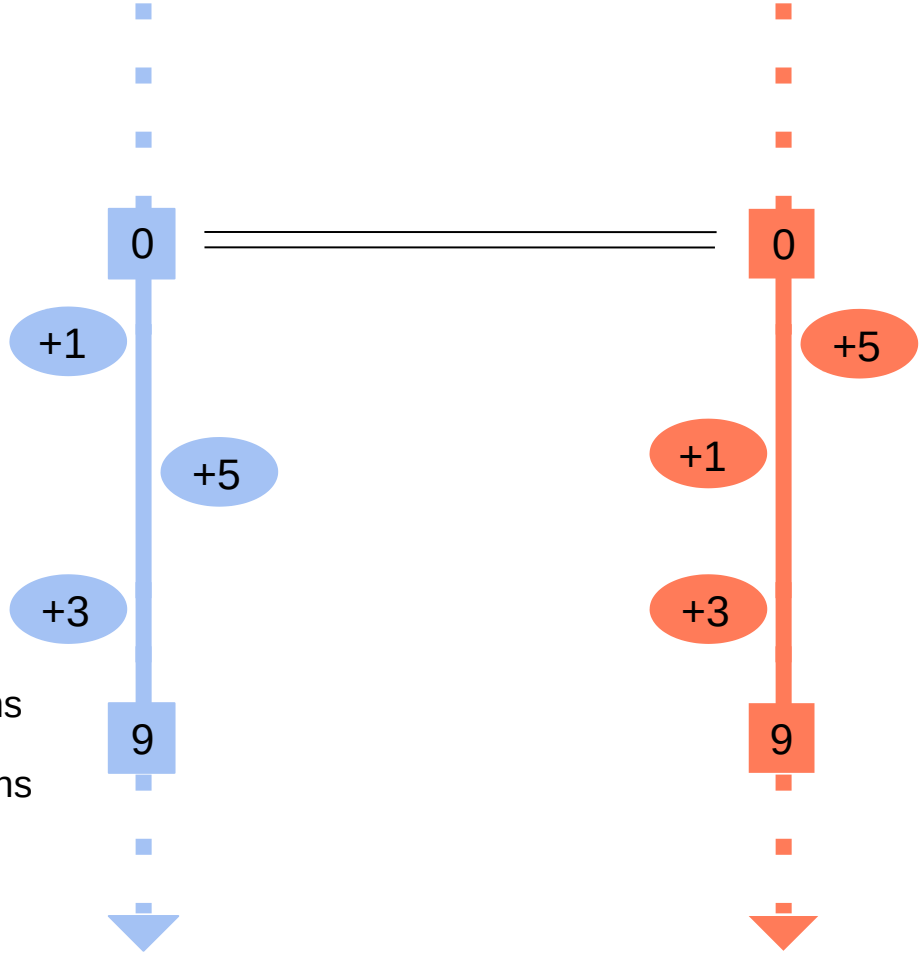

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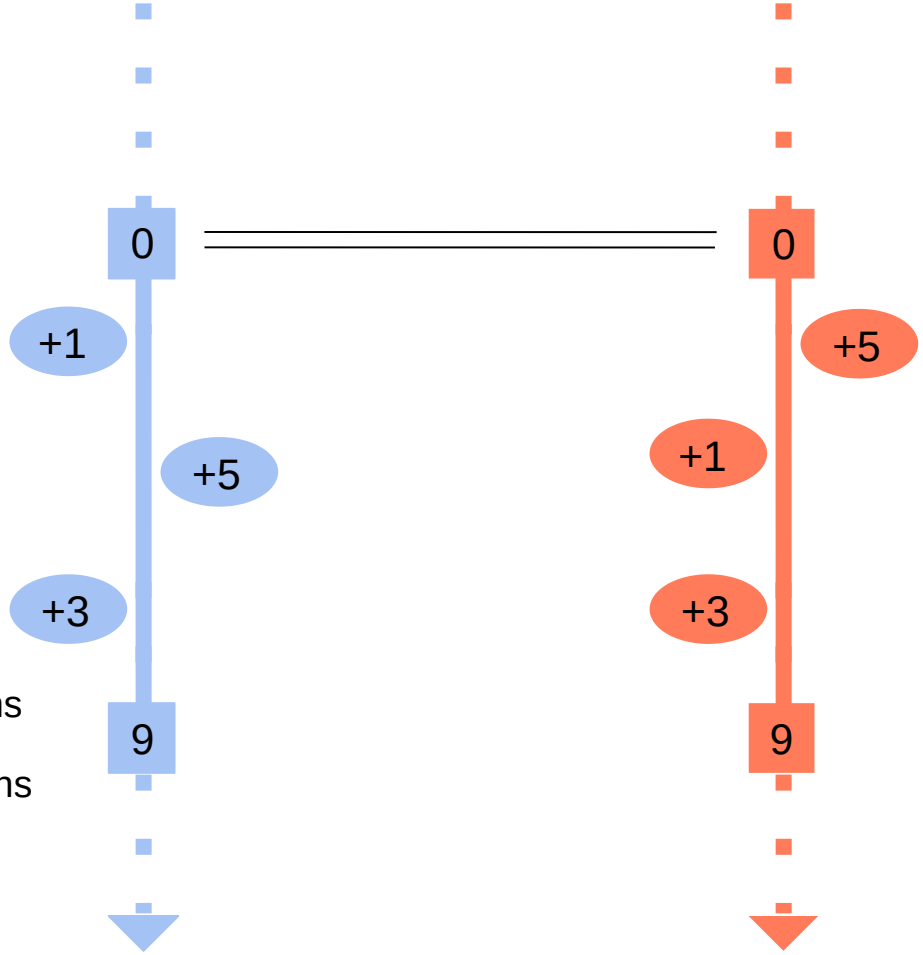

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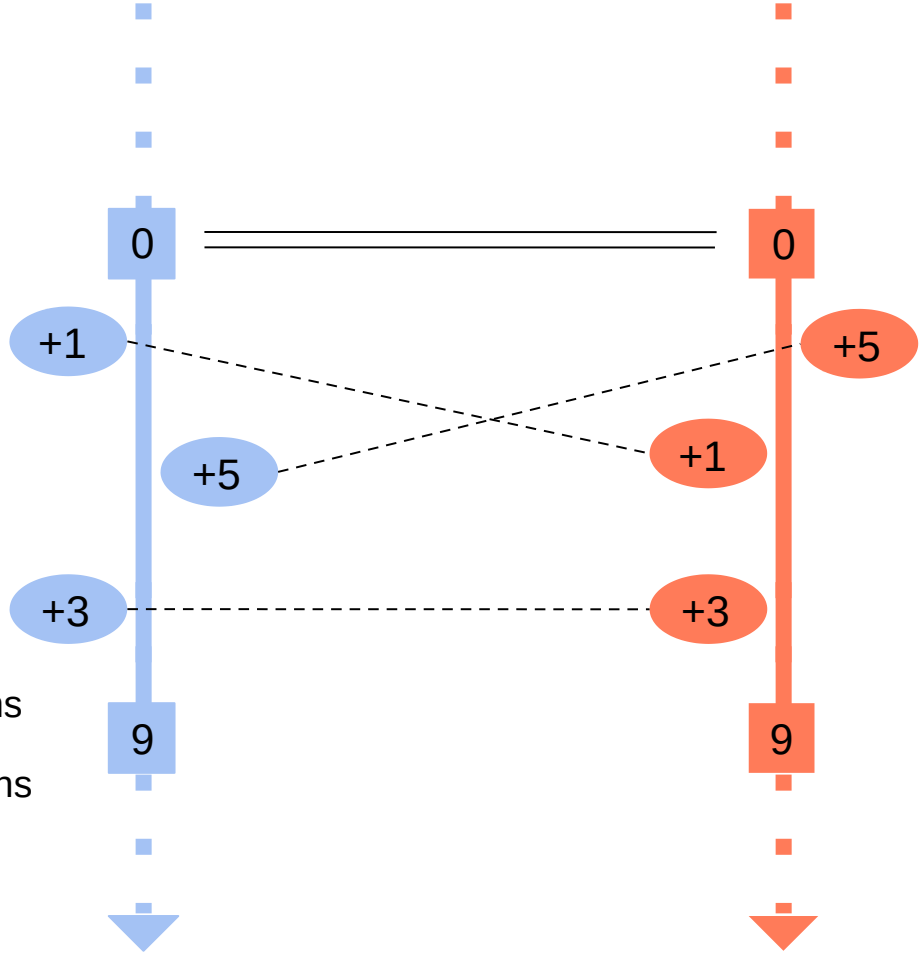

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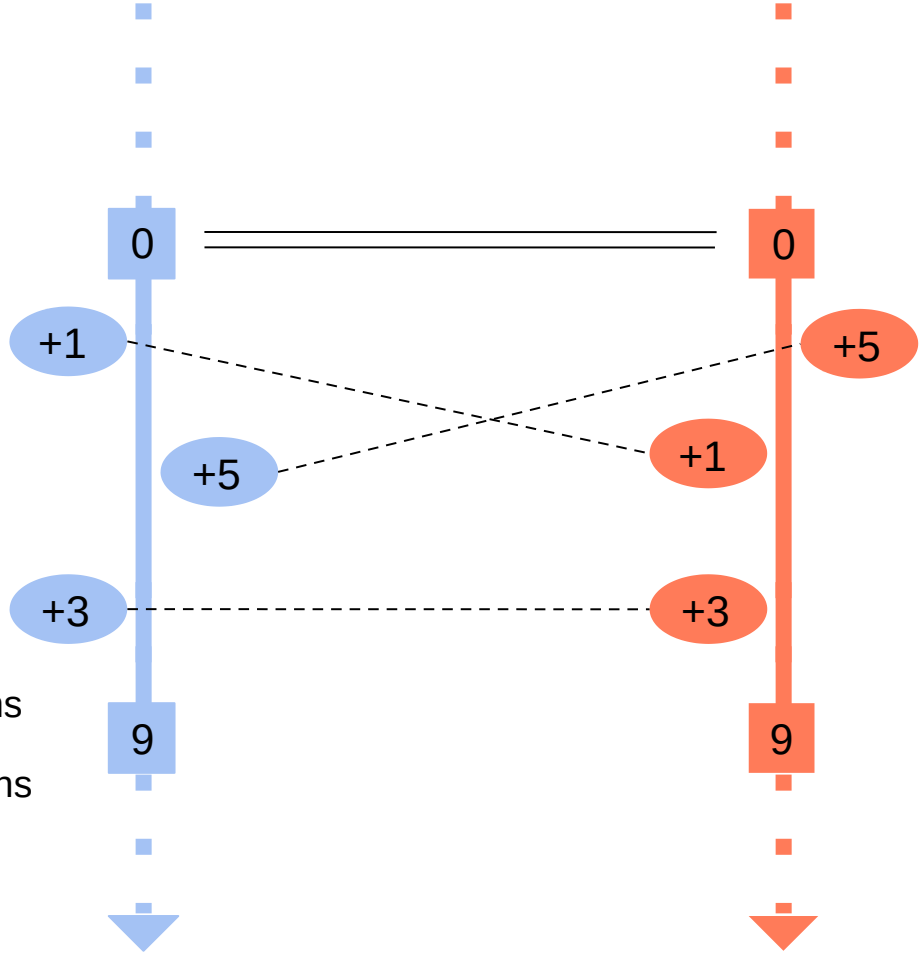

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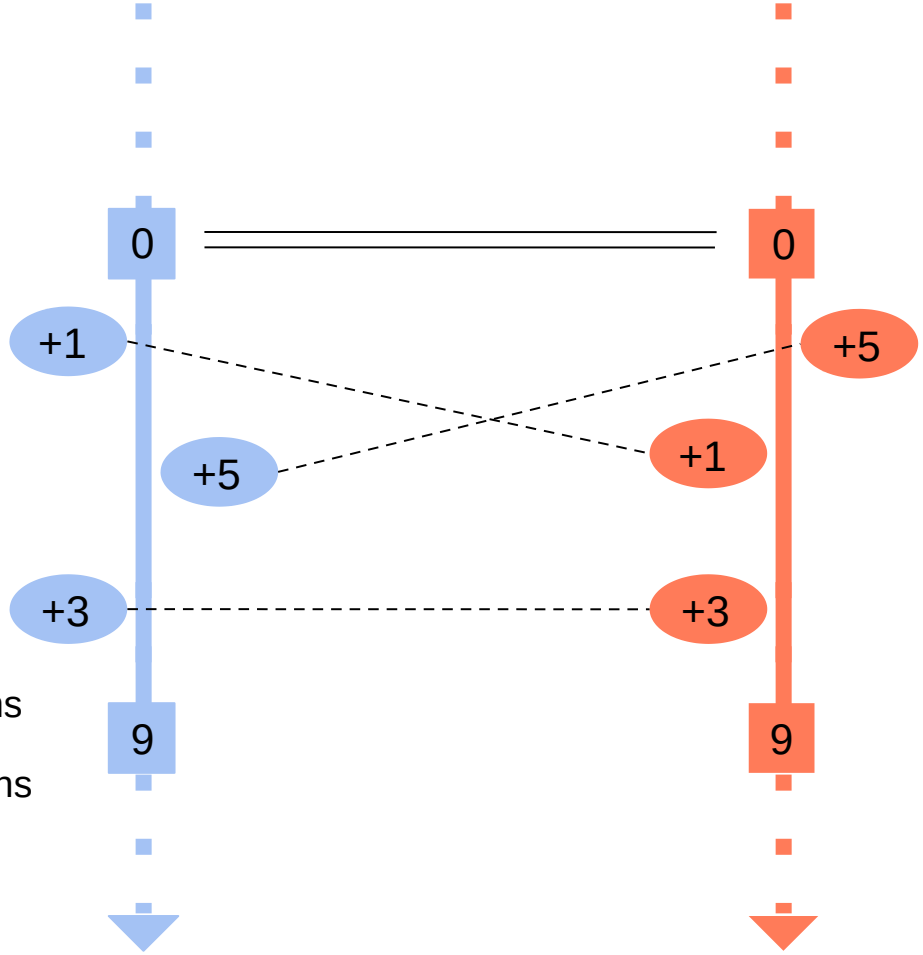

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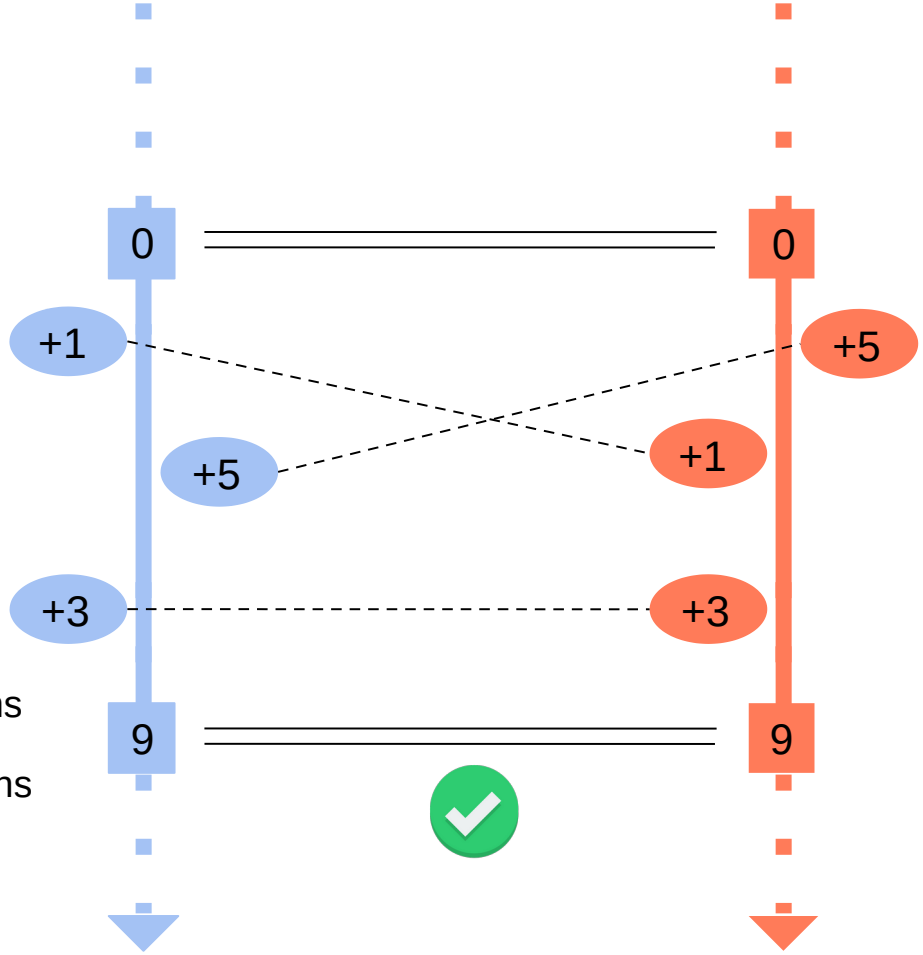

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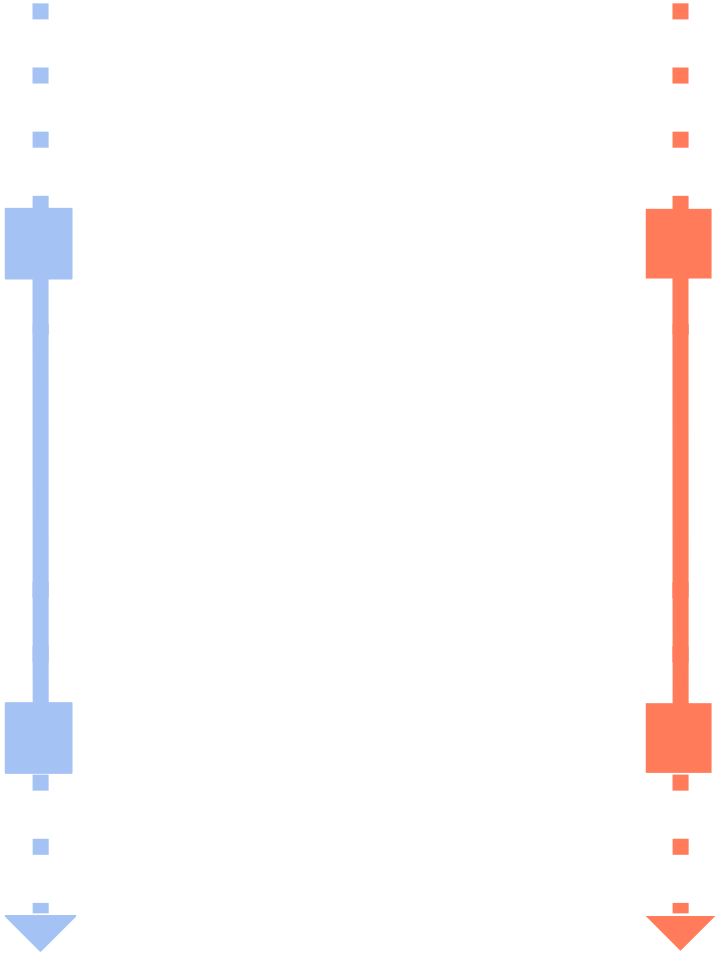

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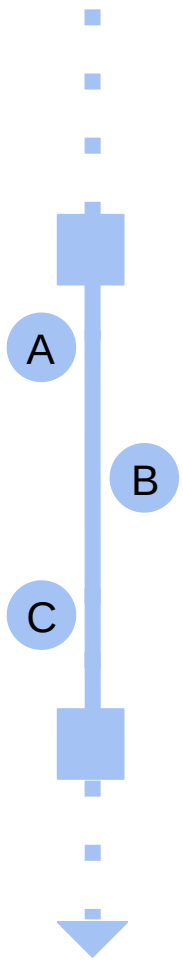

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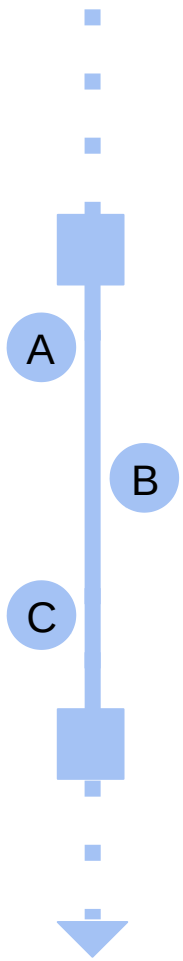

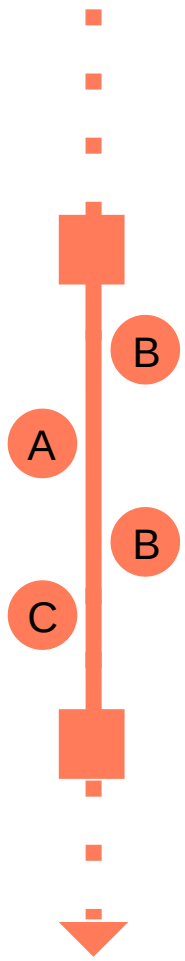

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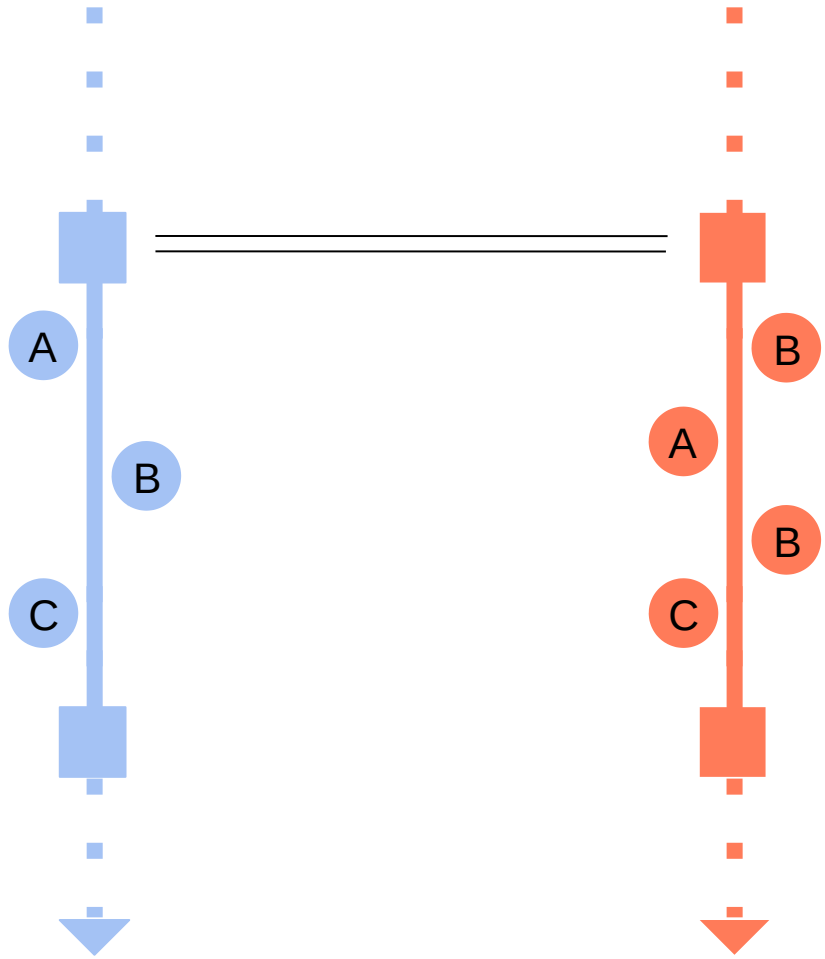

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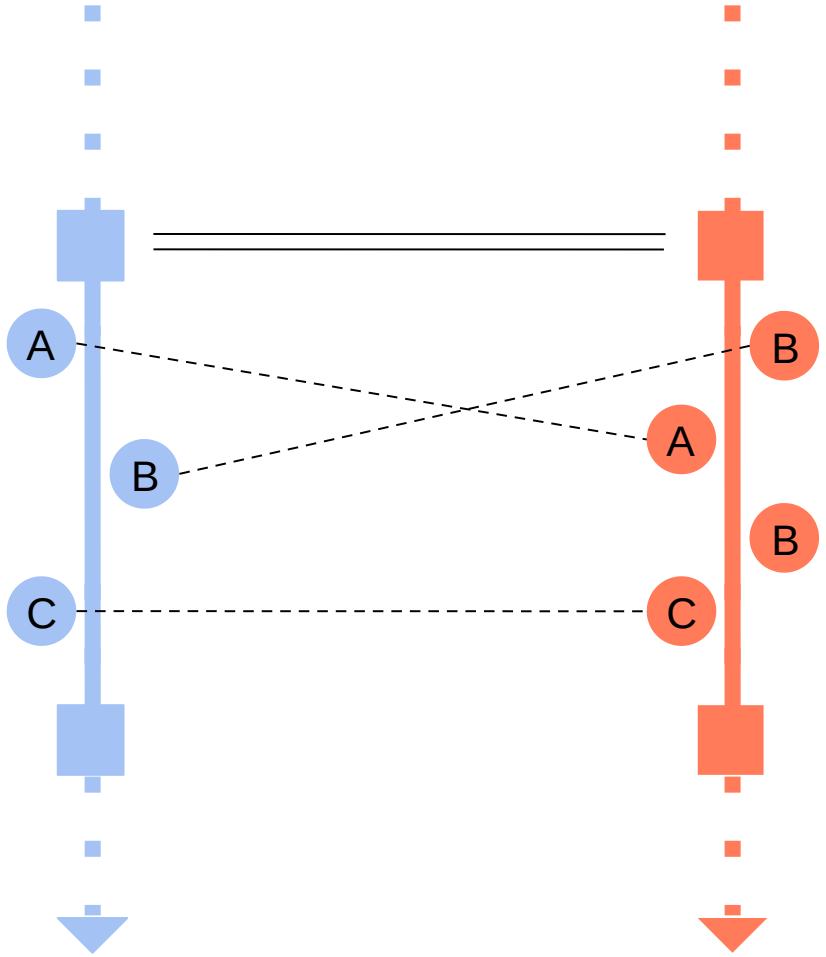

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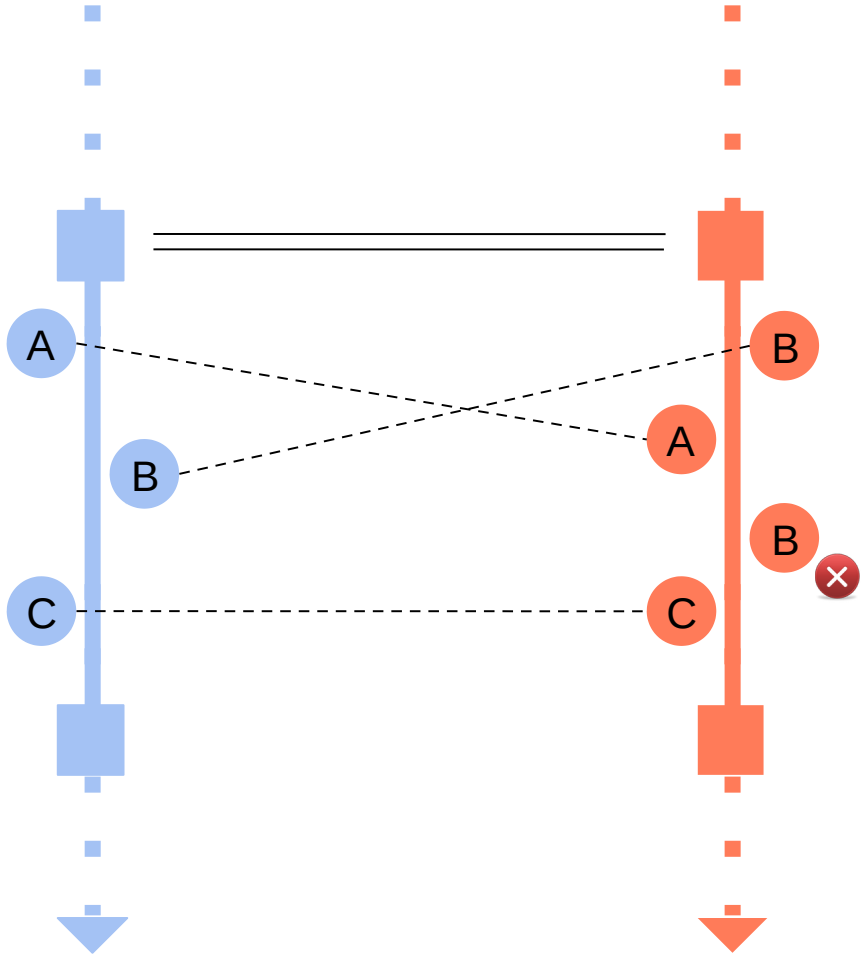

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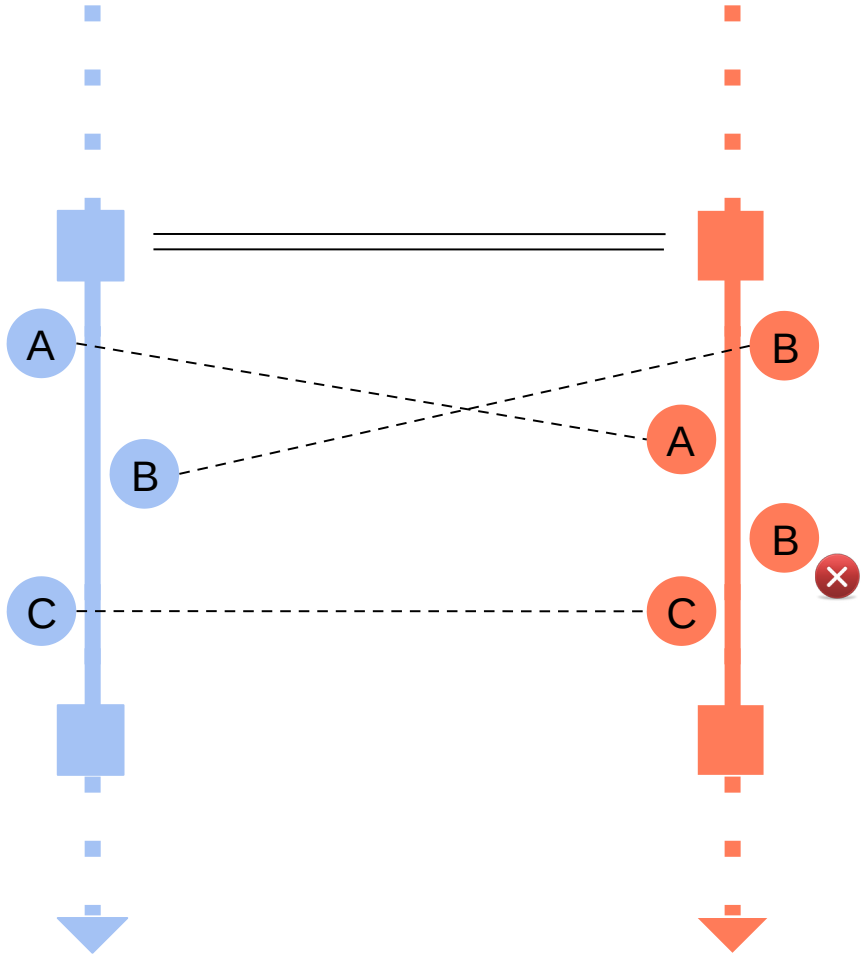

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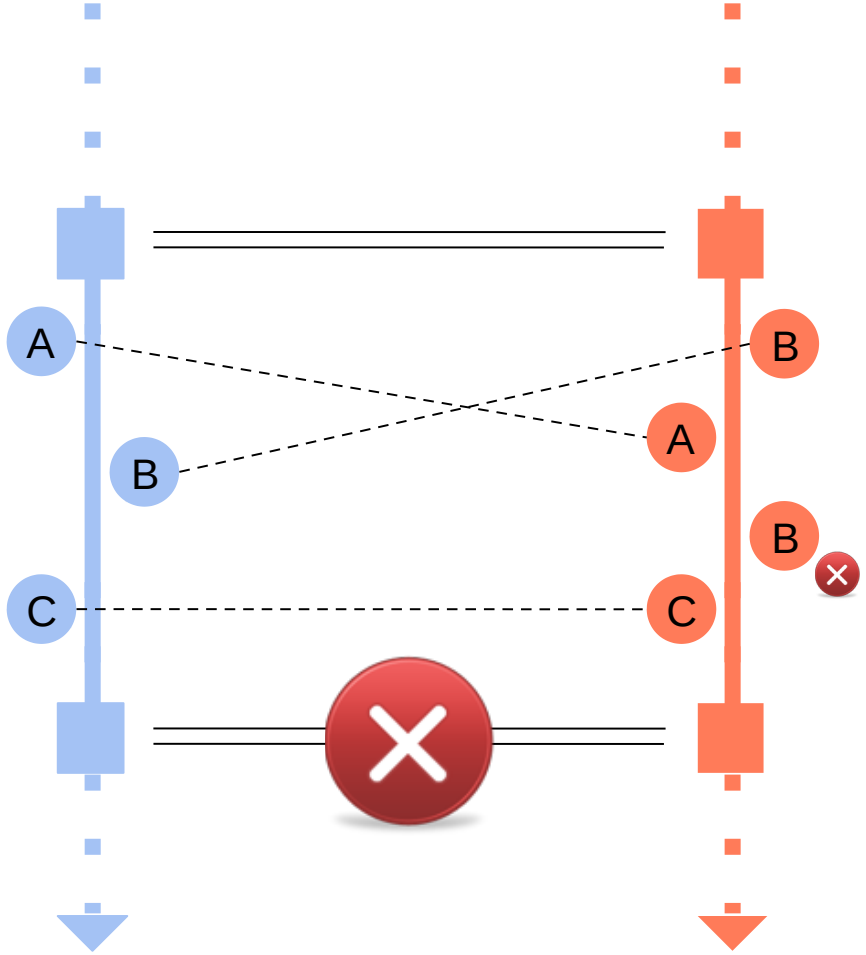

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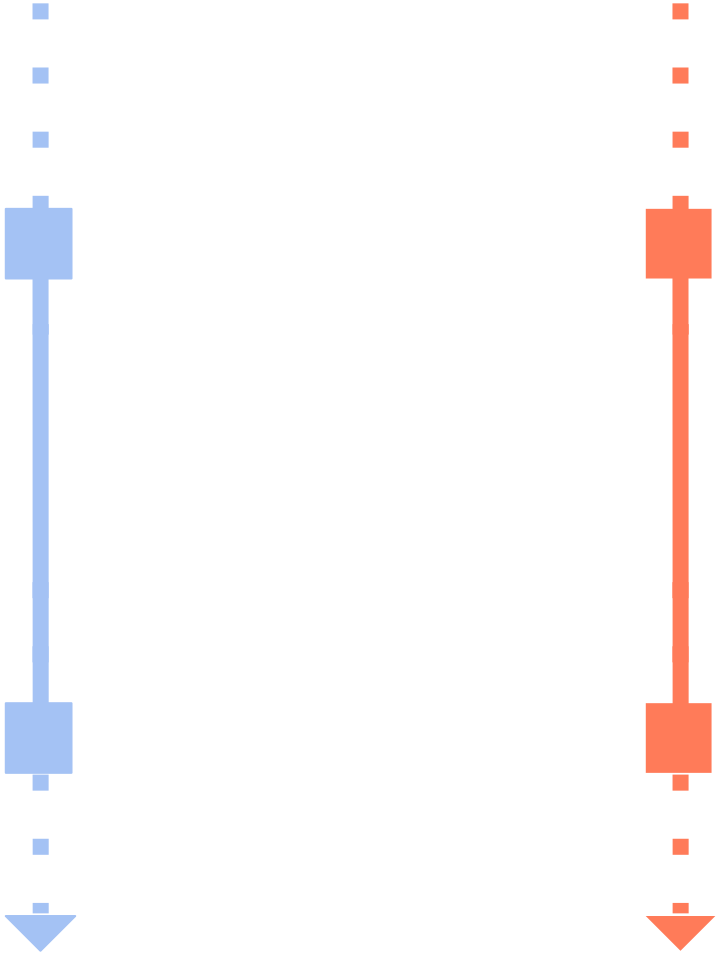

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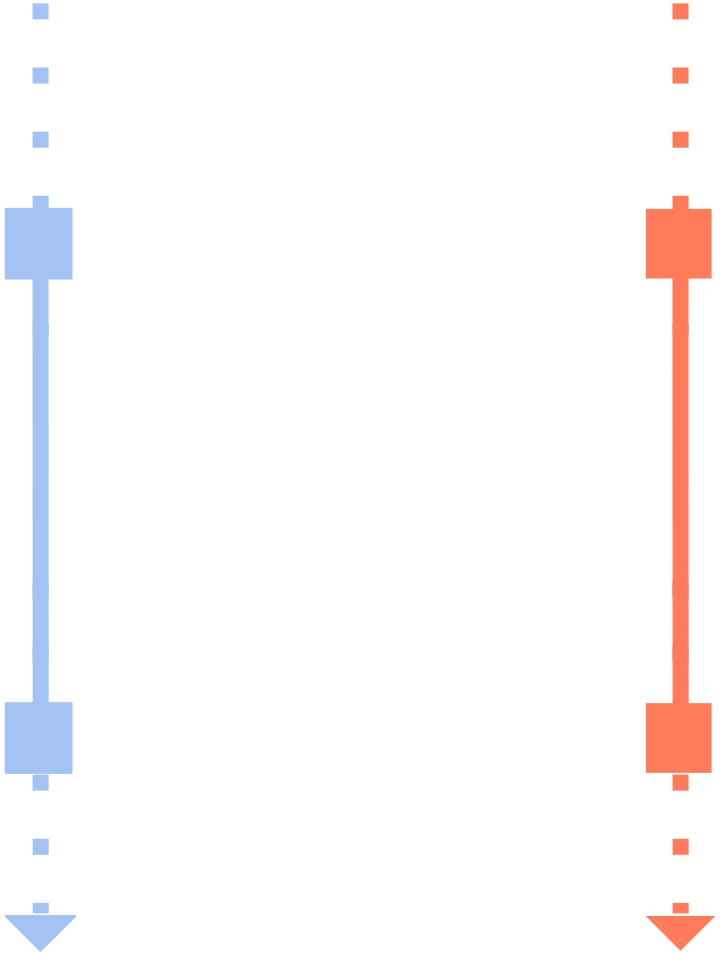

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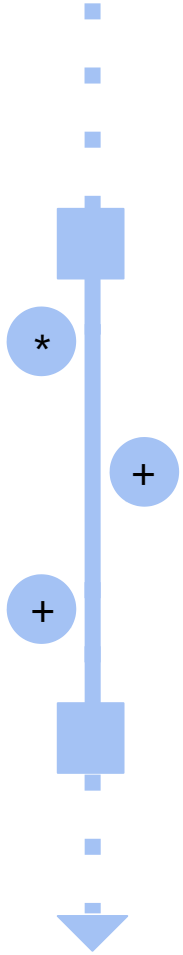

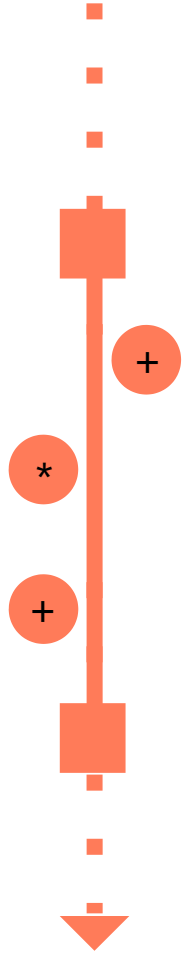

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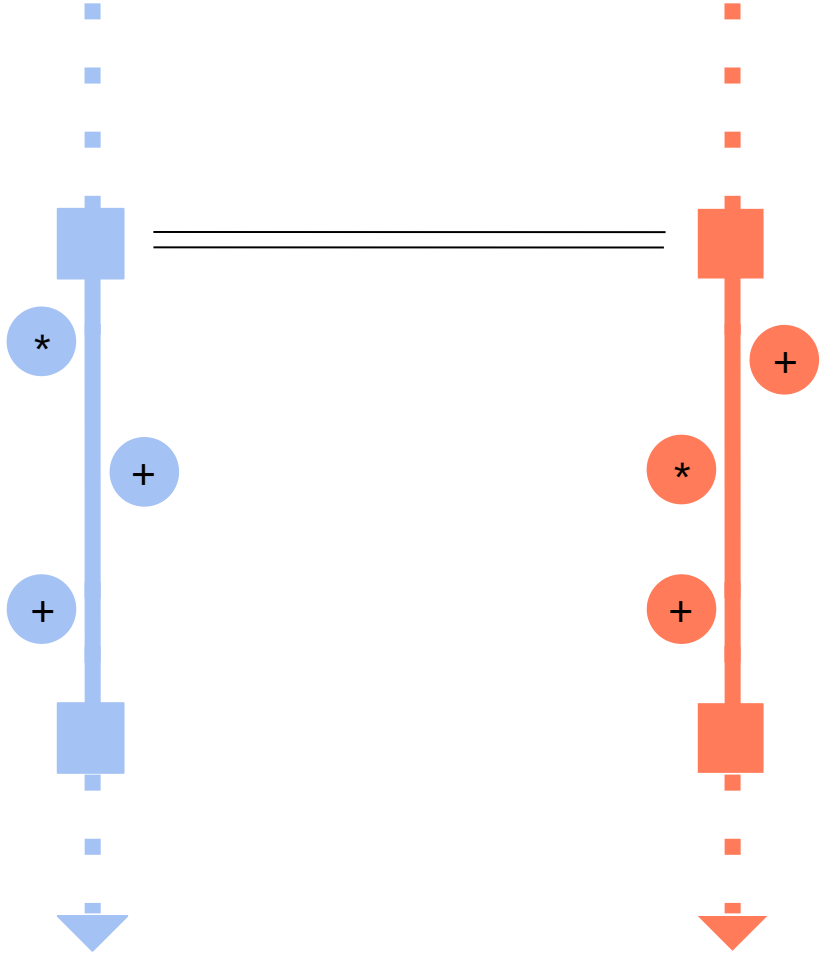

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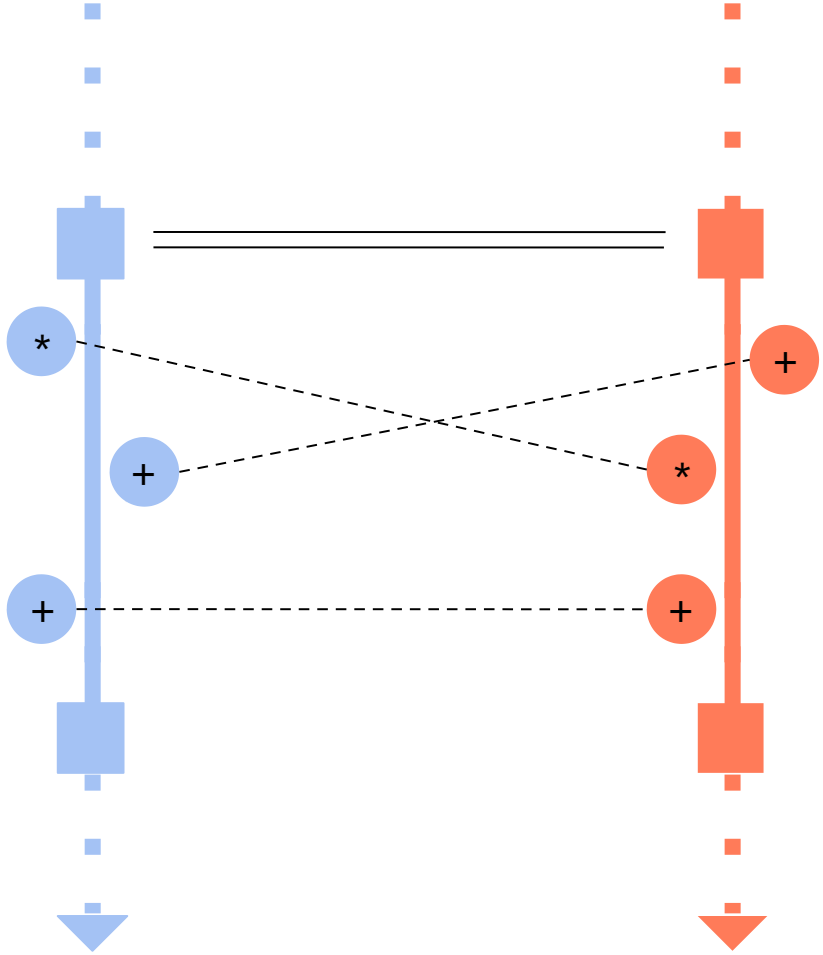

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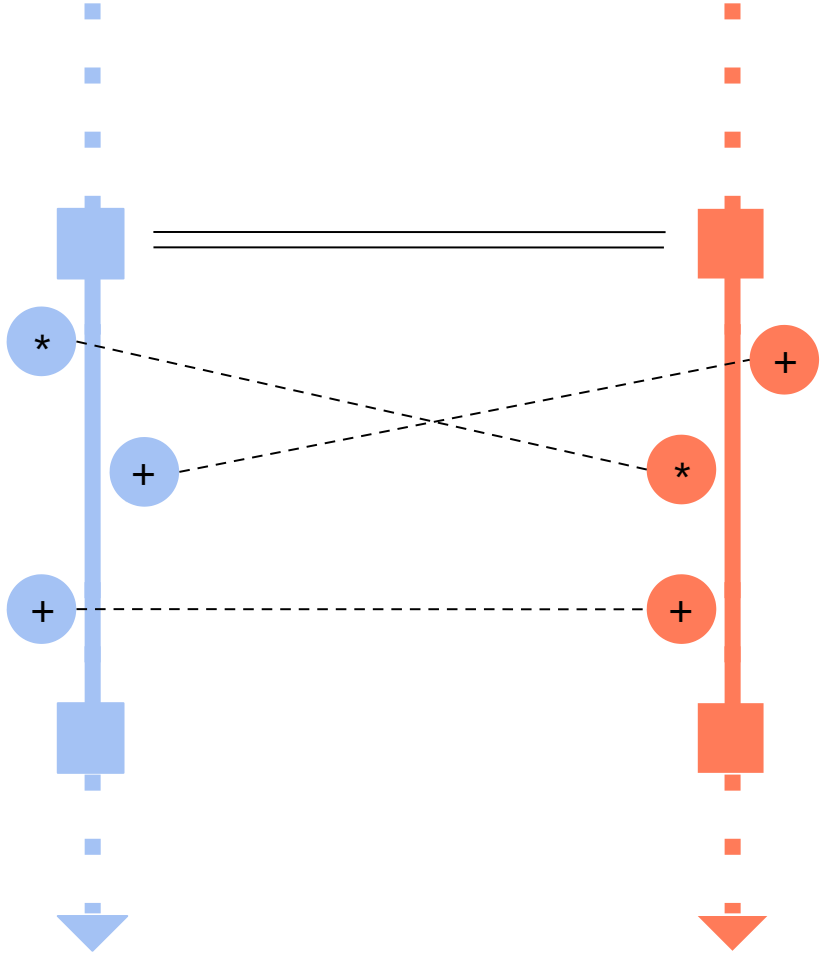

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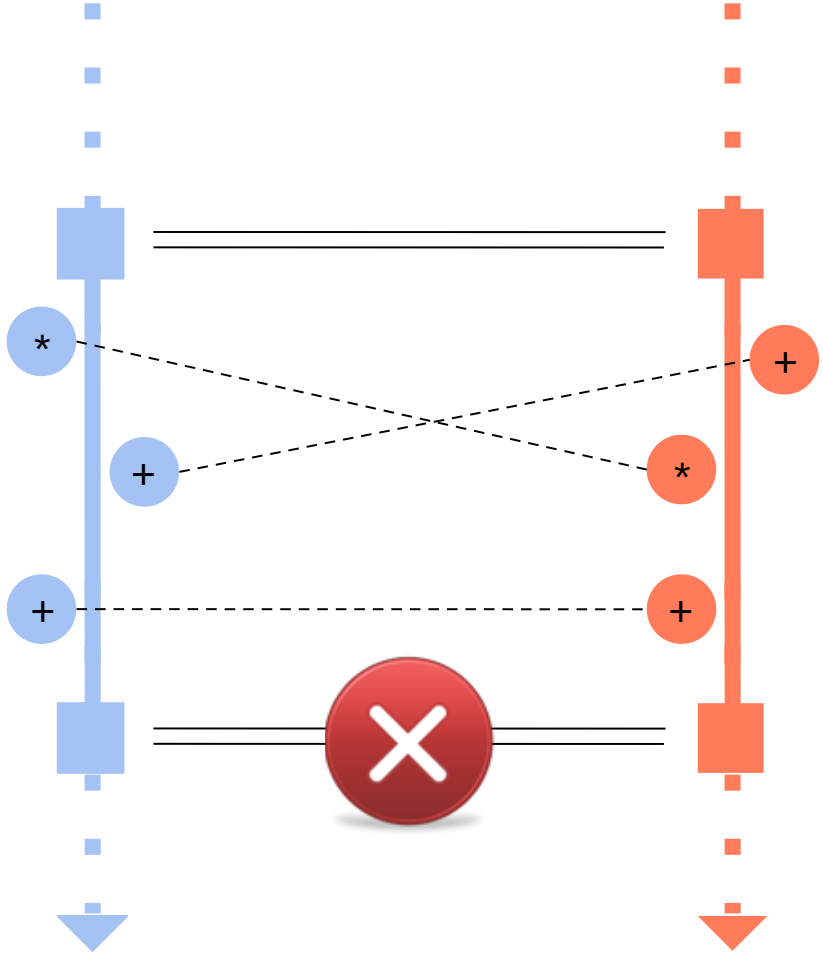

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Back to Program Verification

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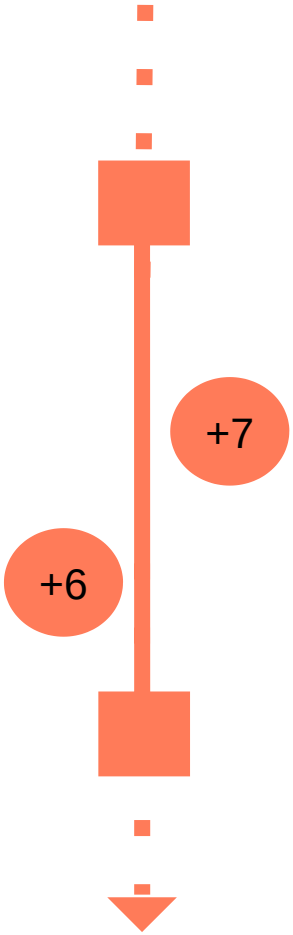
shared = l
share
while i < h:          ||  while j < 100:
  i += 1              ||    j += 1
                      ||
atomic:               ||  atomic:
  shared += 6         ||    shared += 7
unshare
return shared
```



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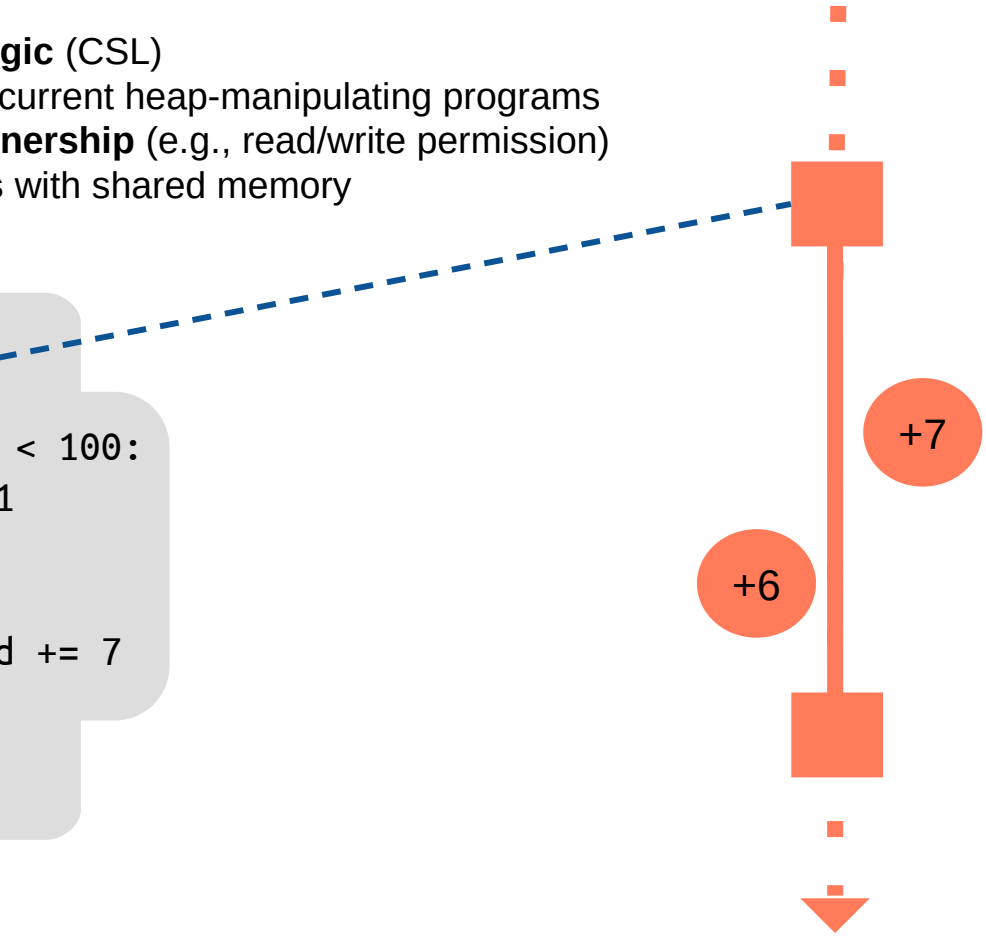

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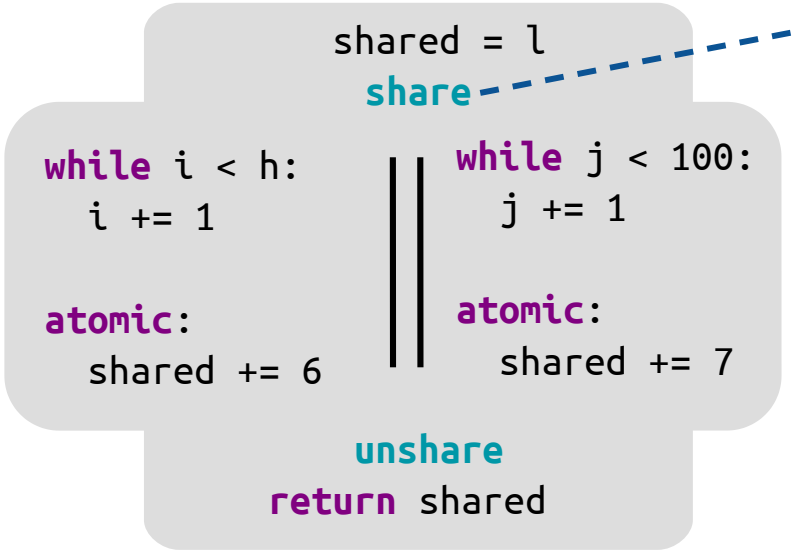

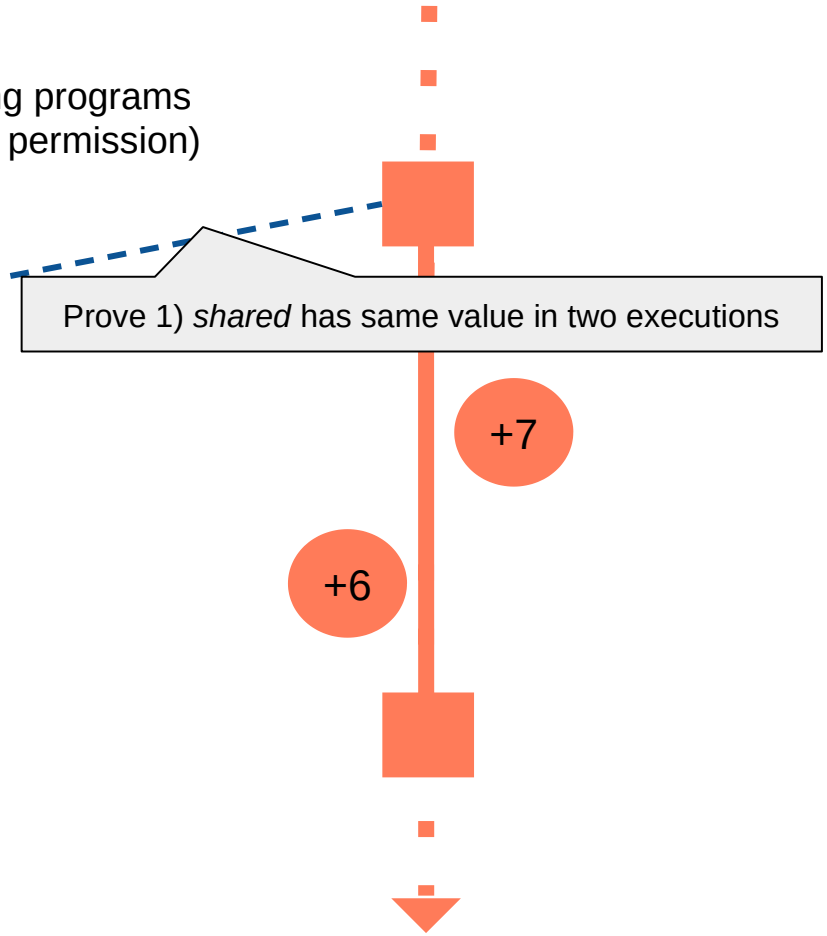

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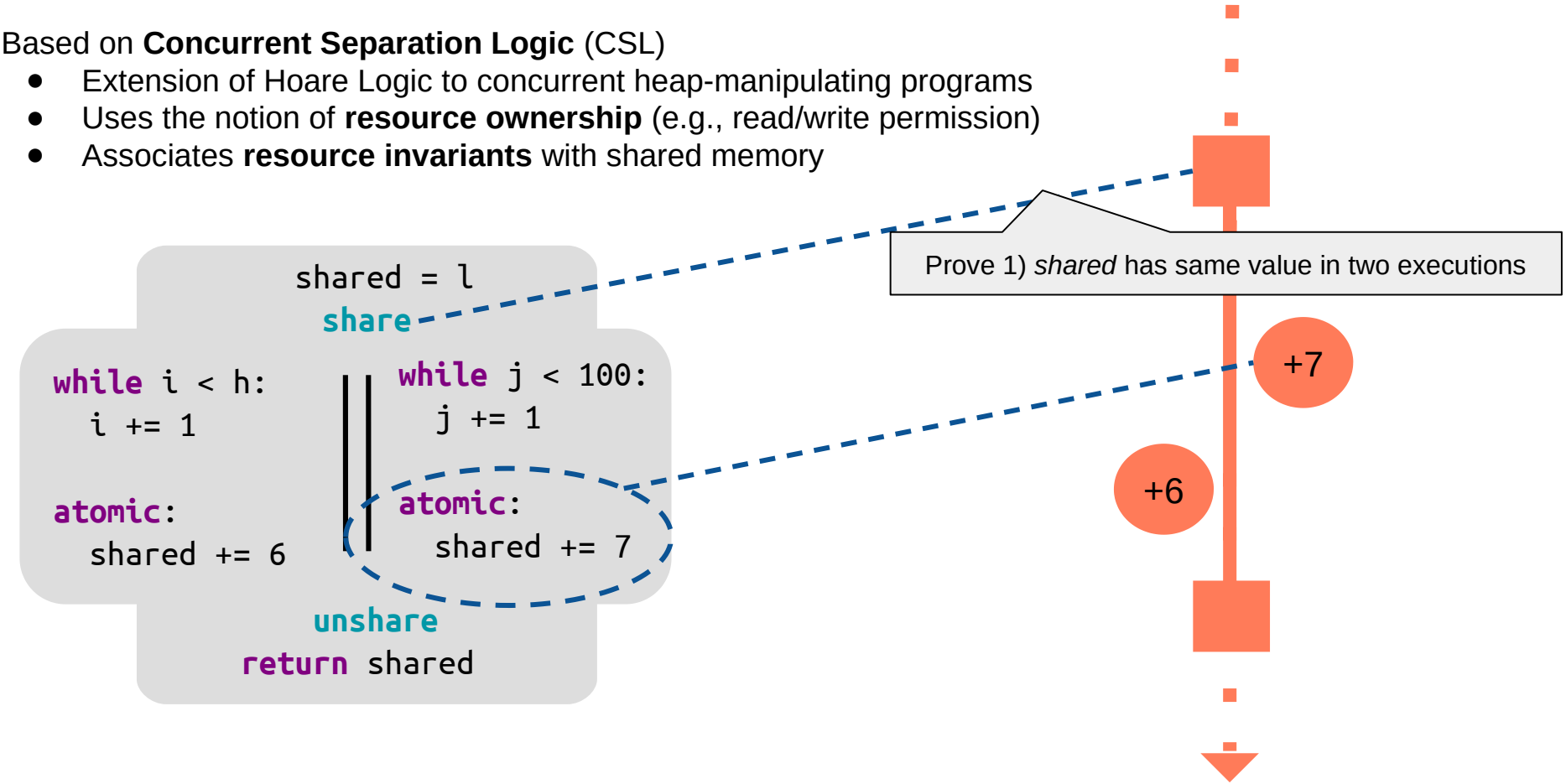

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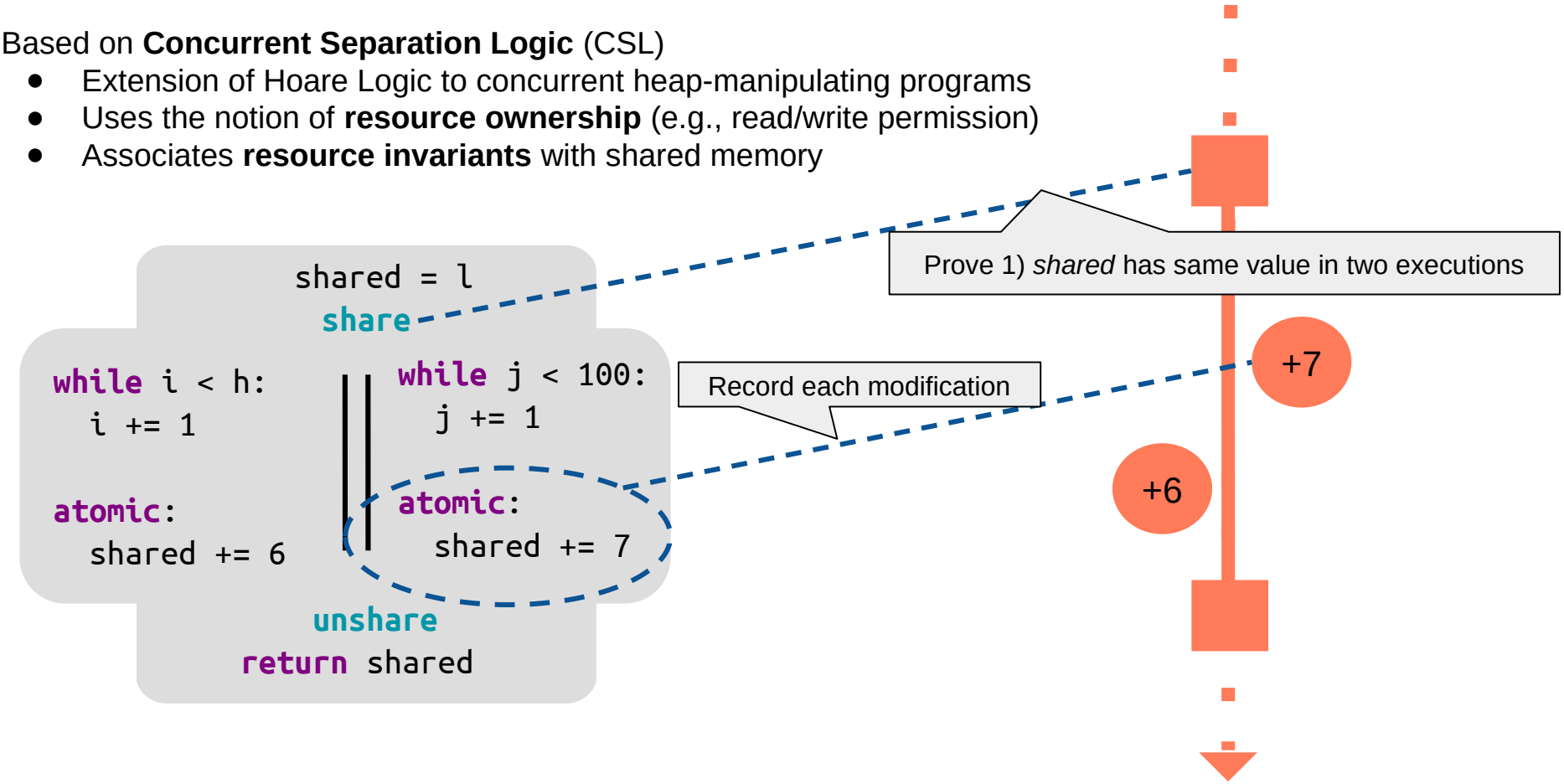

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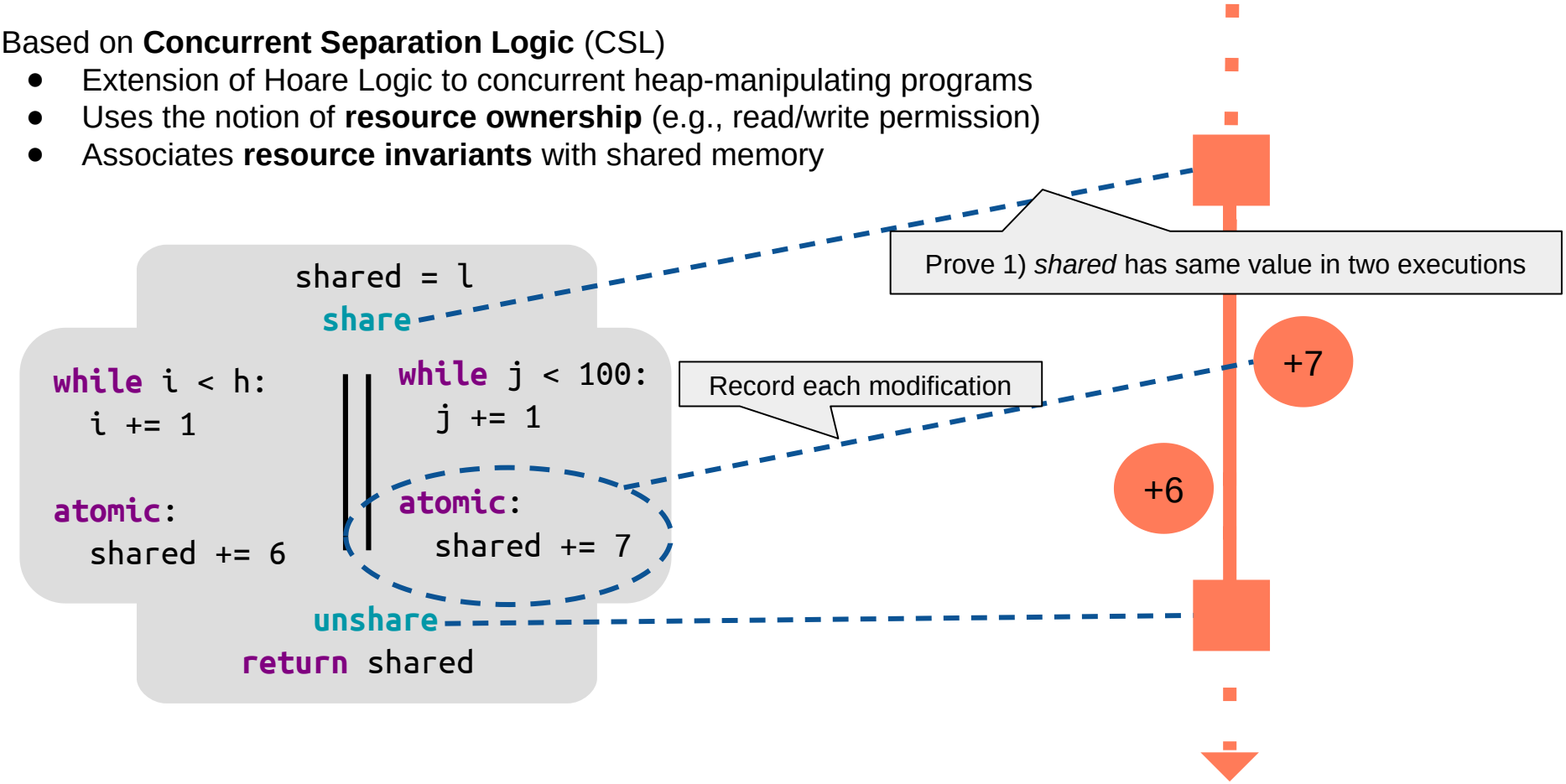

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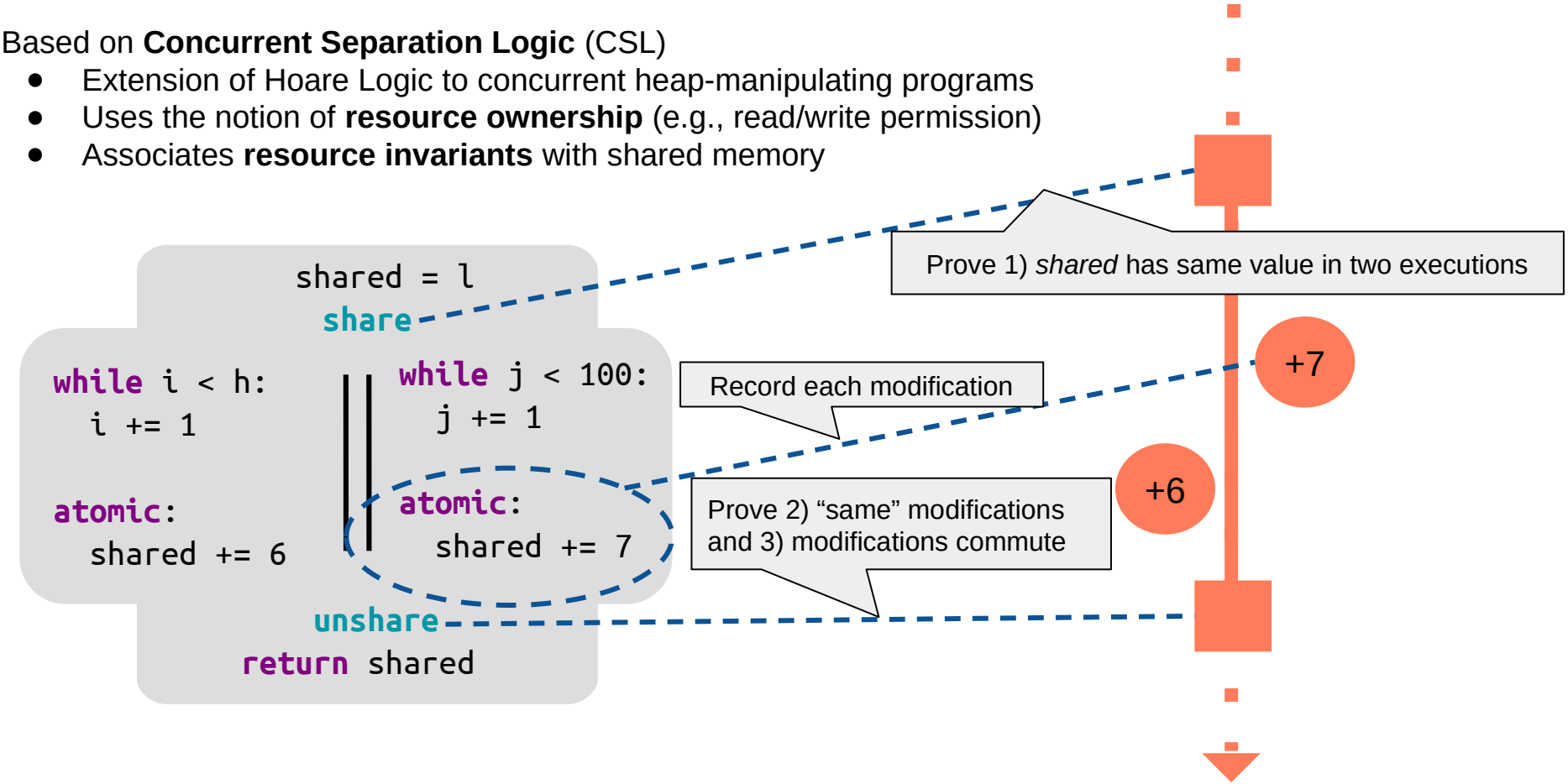

Verification Approach

Based on **Concurrent Separation Logic** (CSL)

- Extension of Hoare Logic to concurrent heap-manipulating programs
- Uses the notion of **resource ownership** (e.g., read/write permission)
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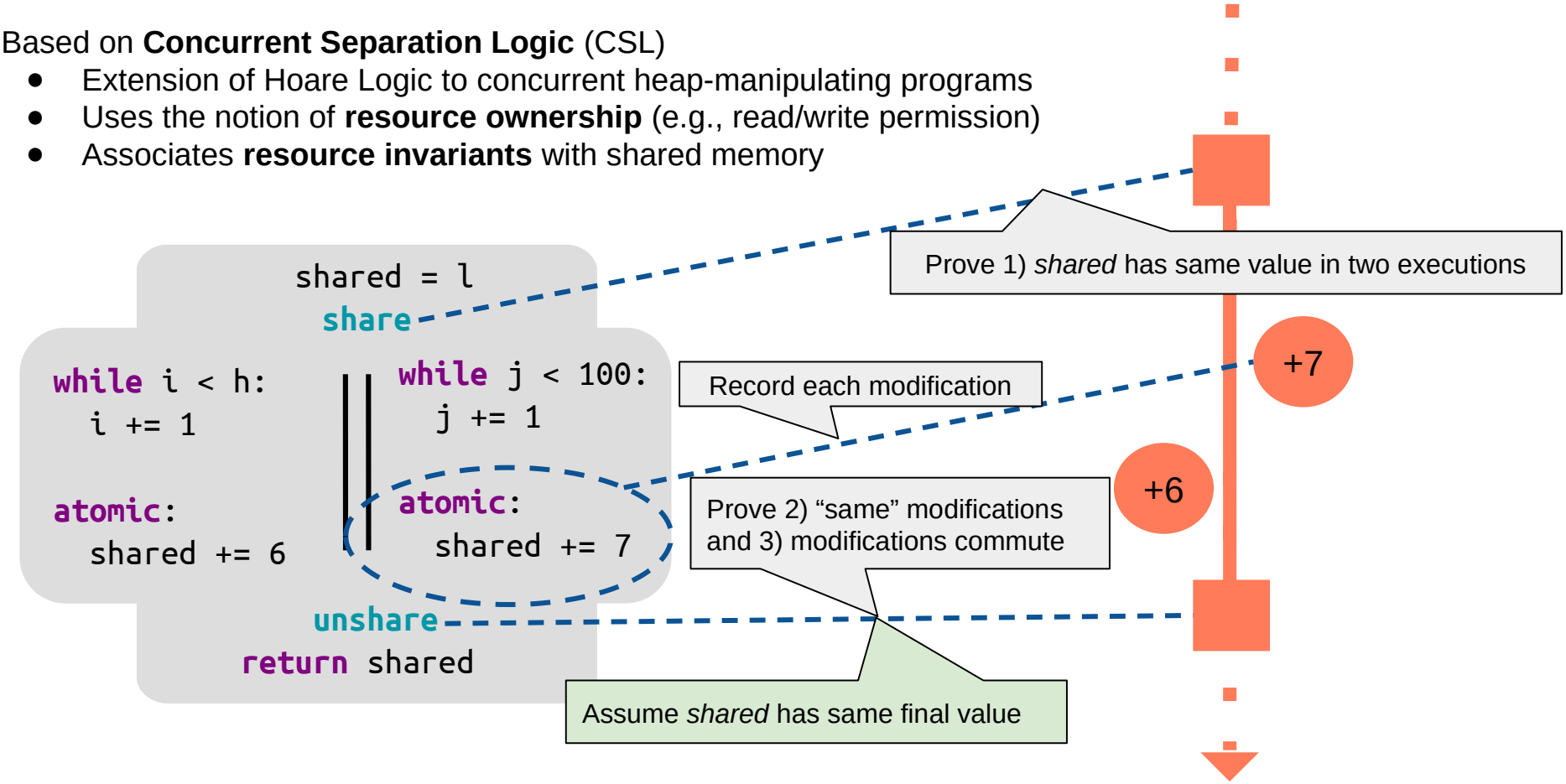

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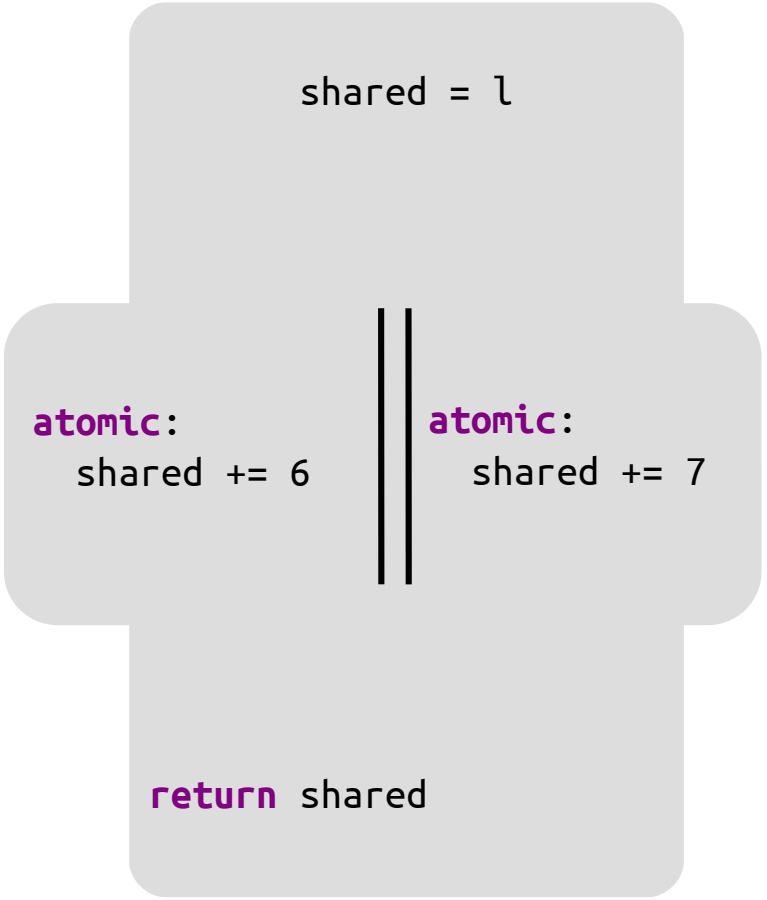

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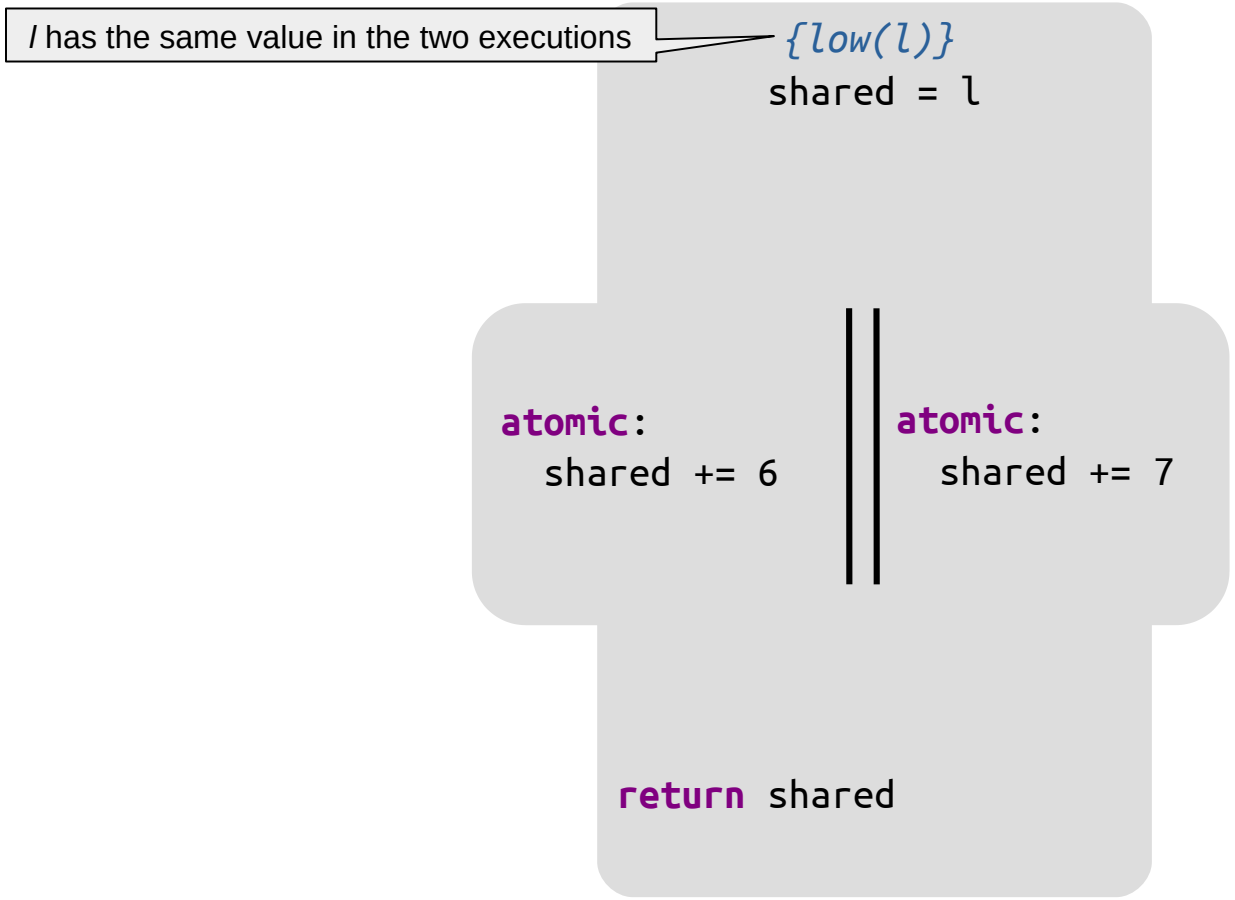

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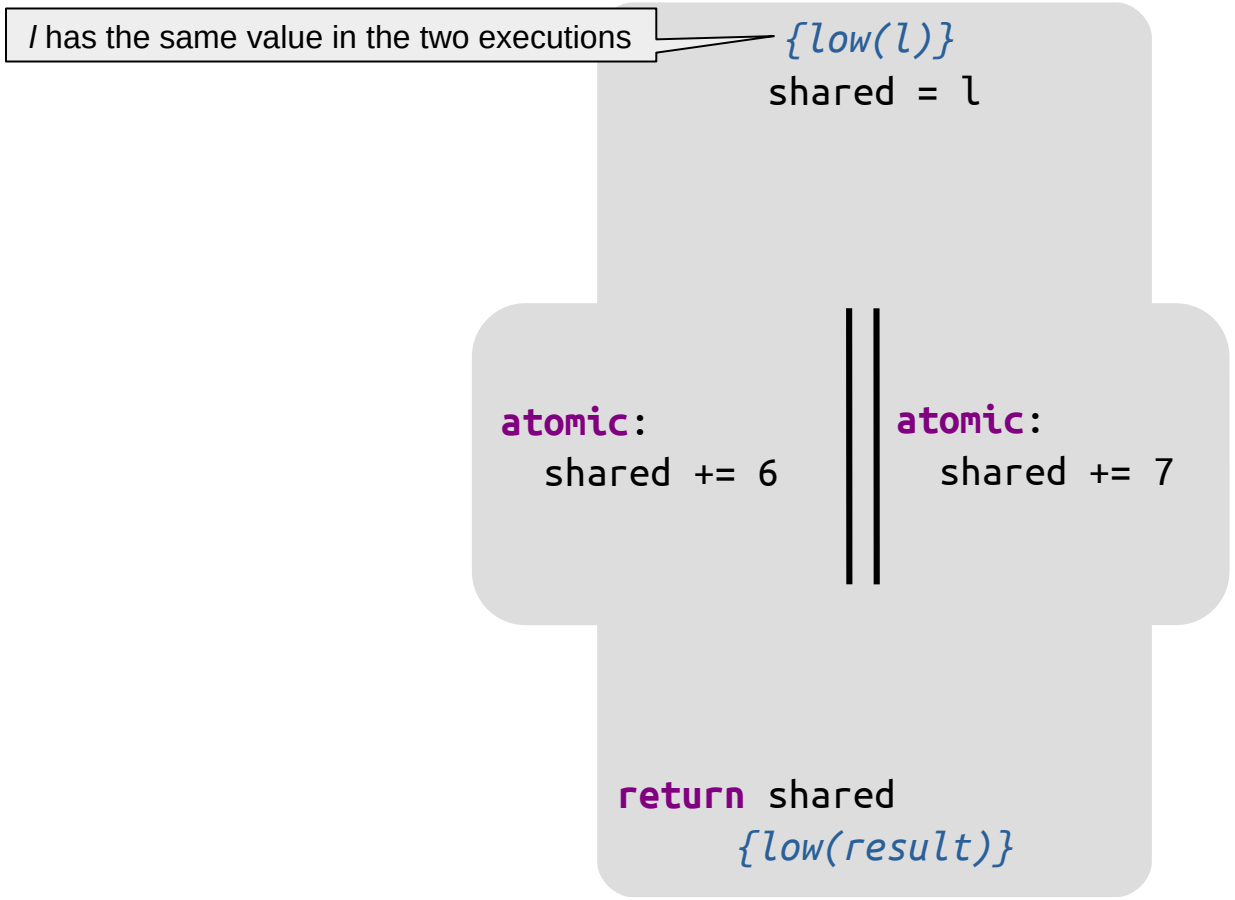

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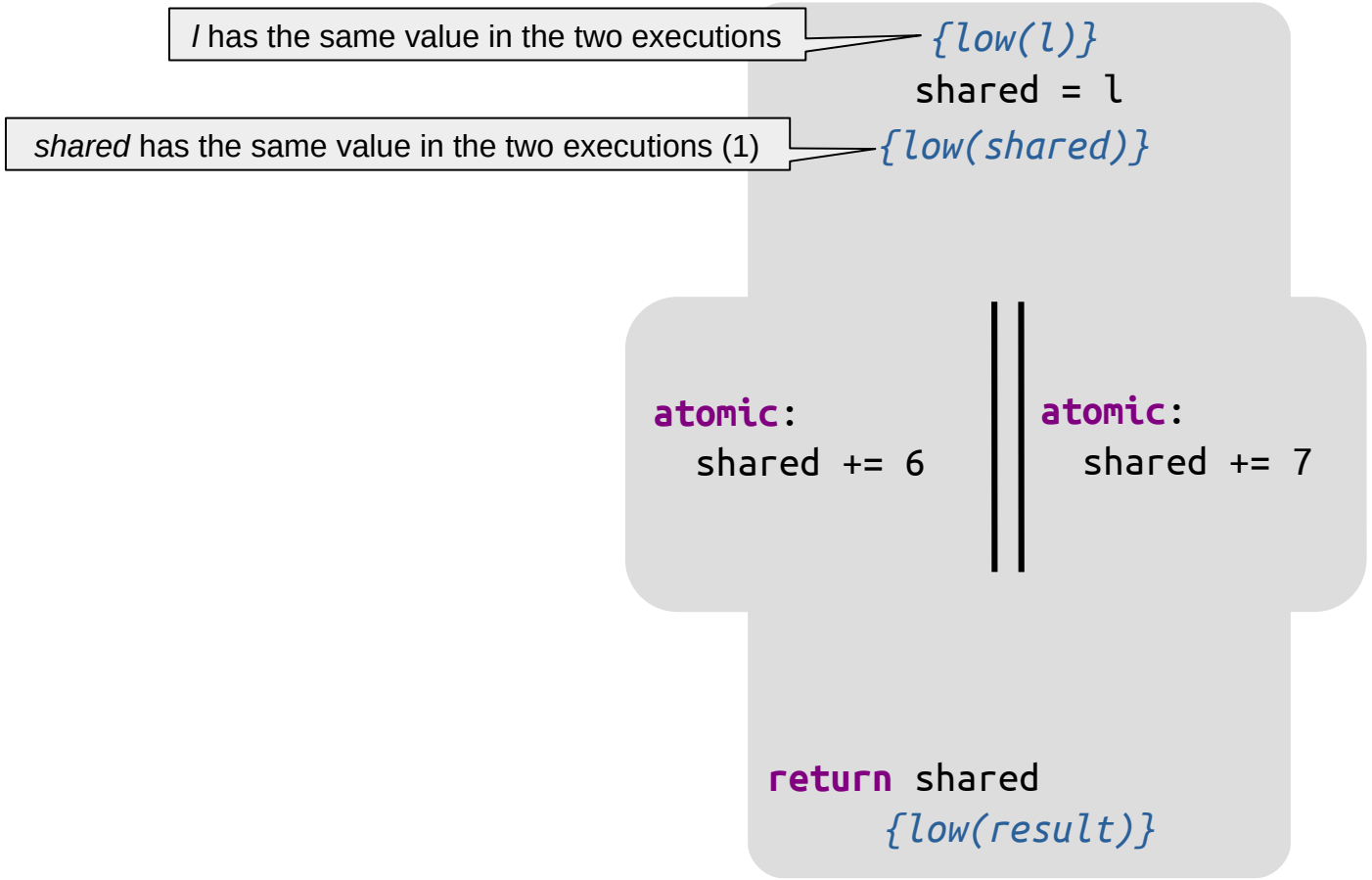

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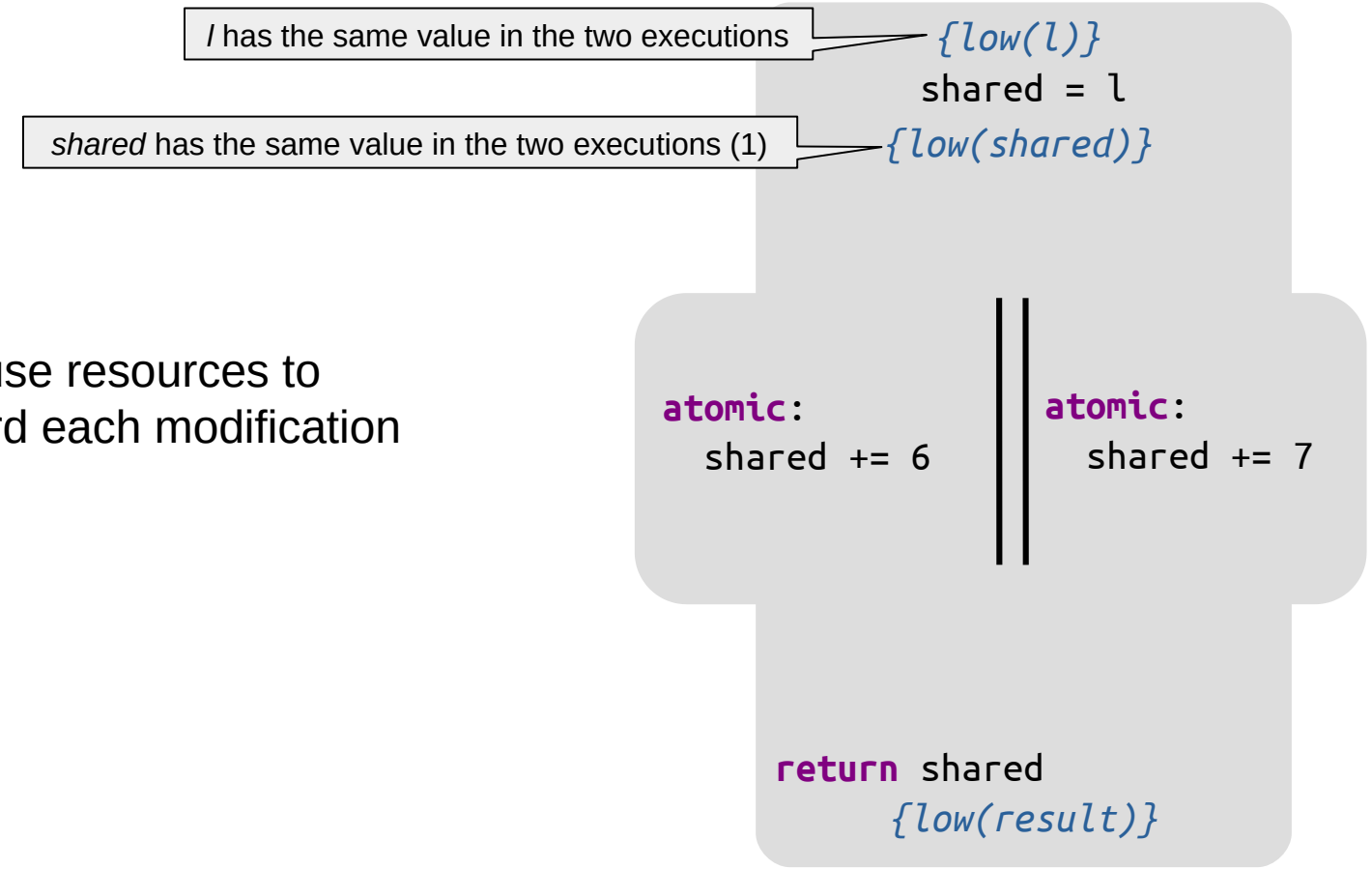
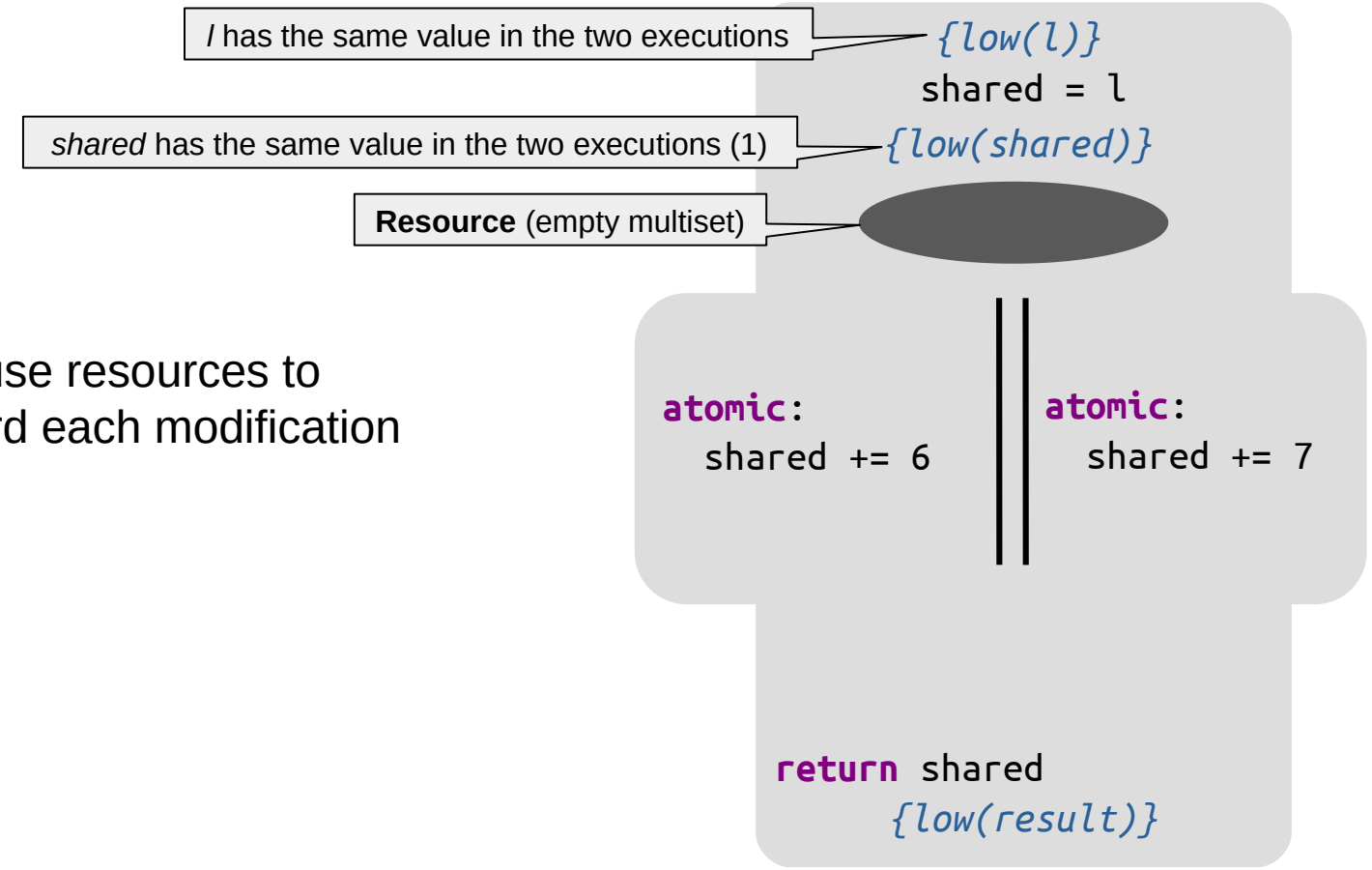


We use resources to record each modification

Verification Approach



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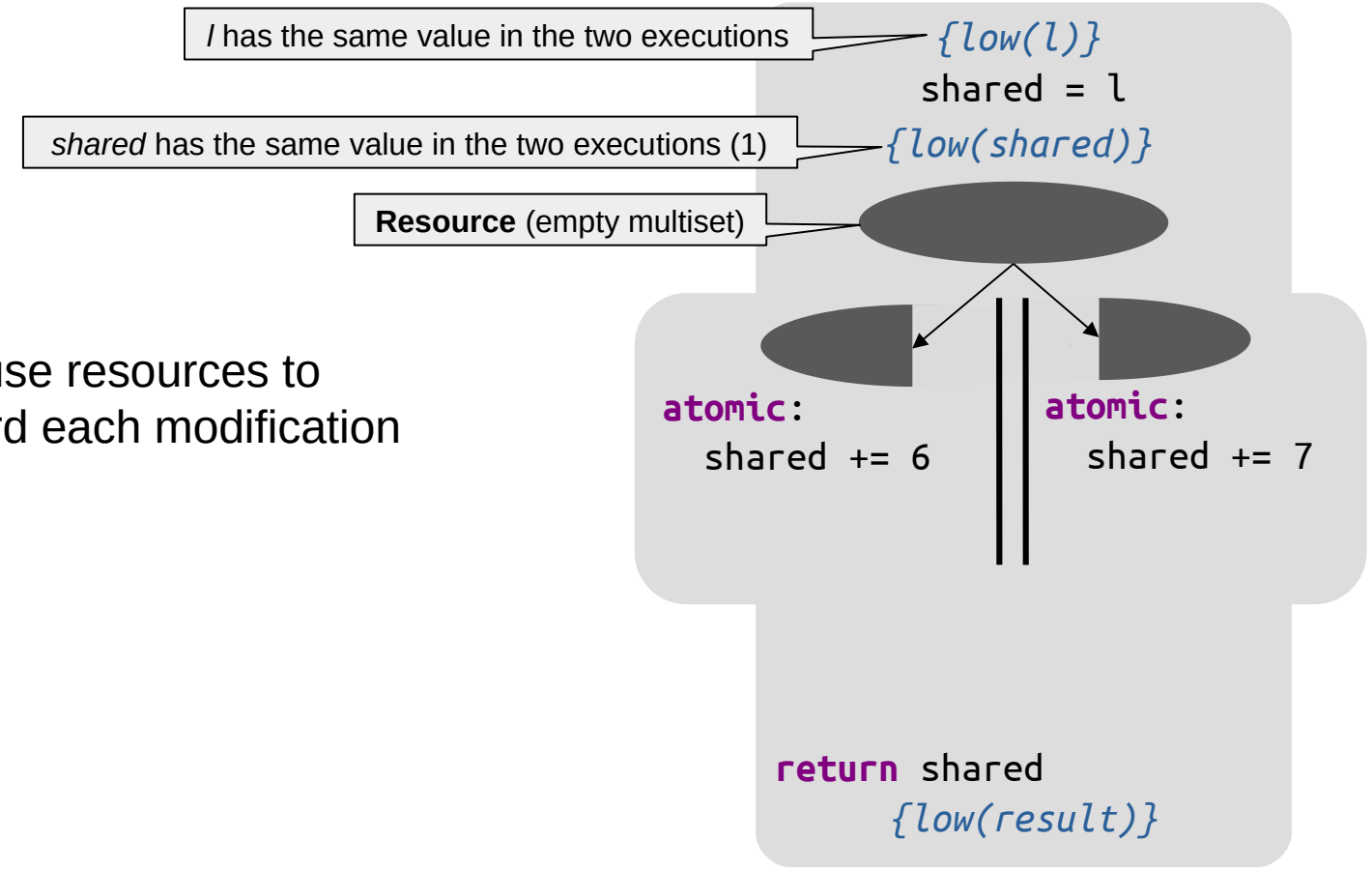

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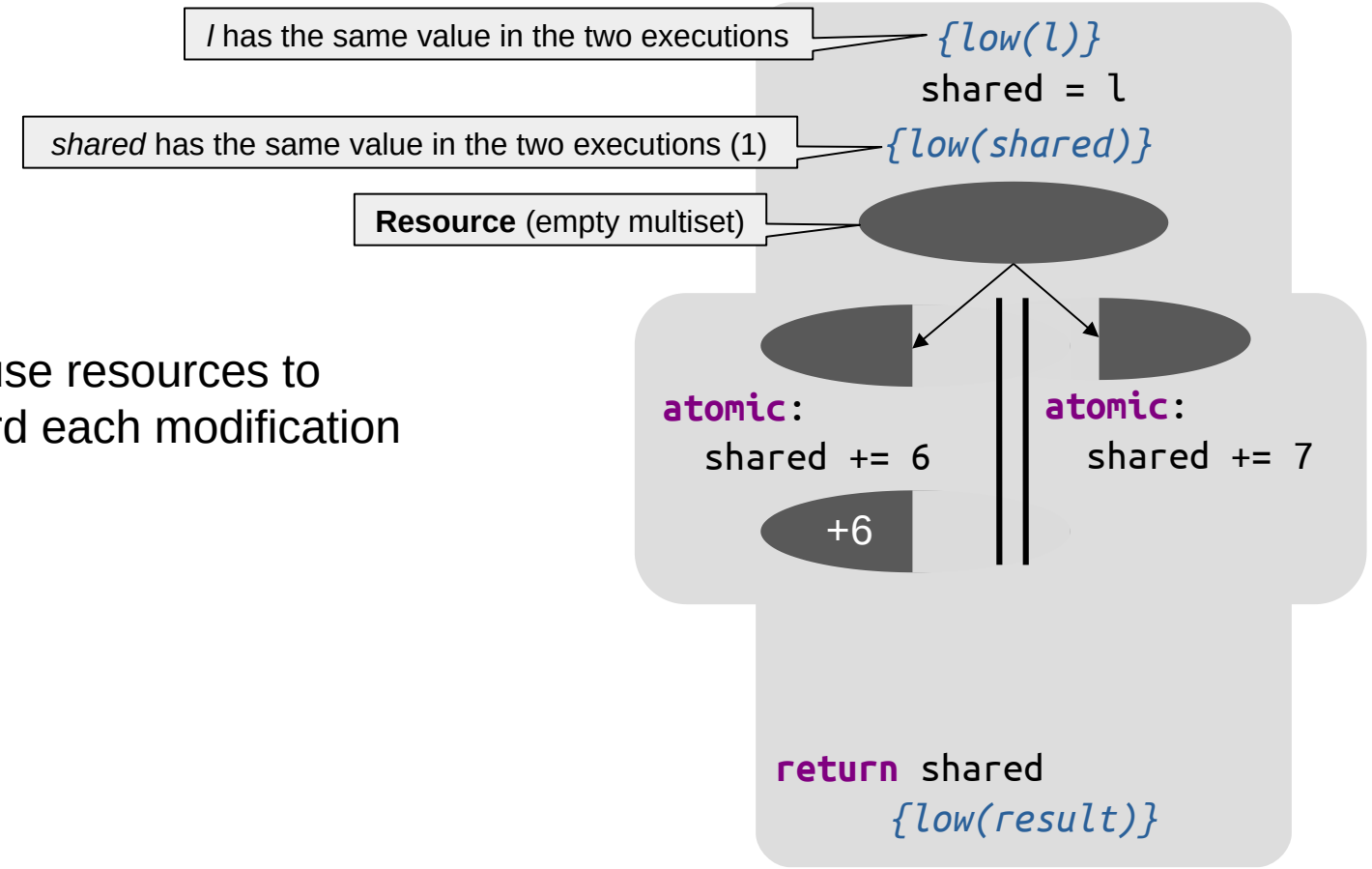

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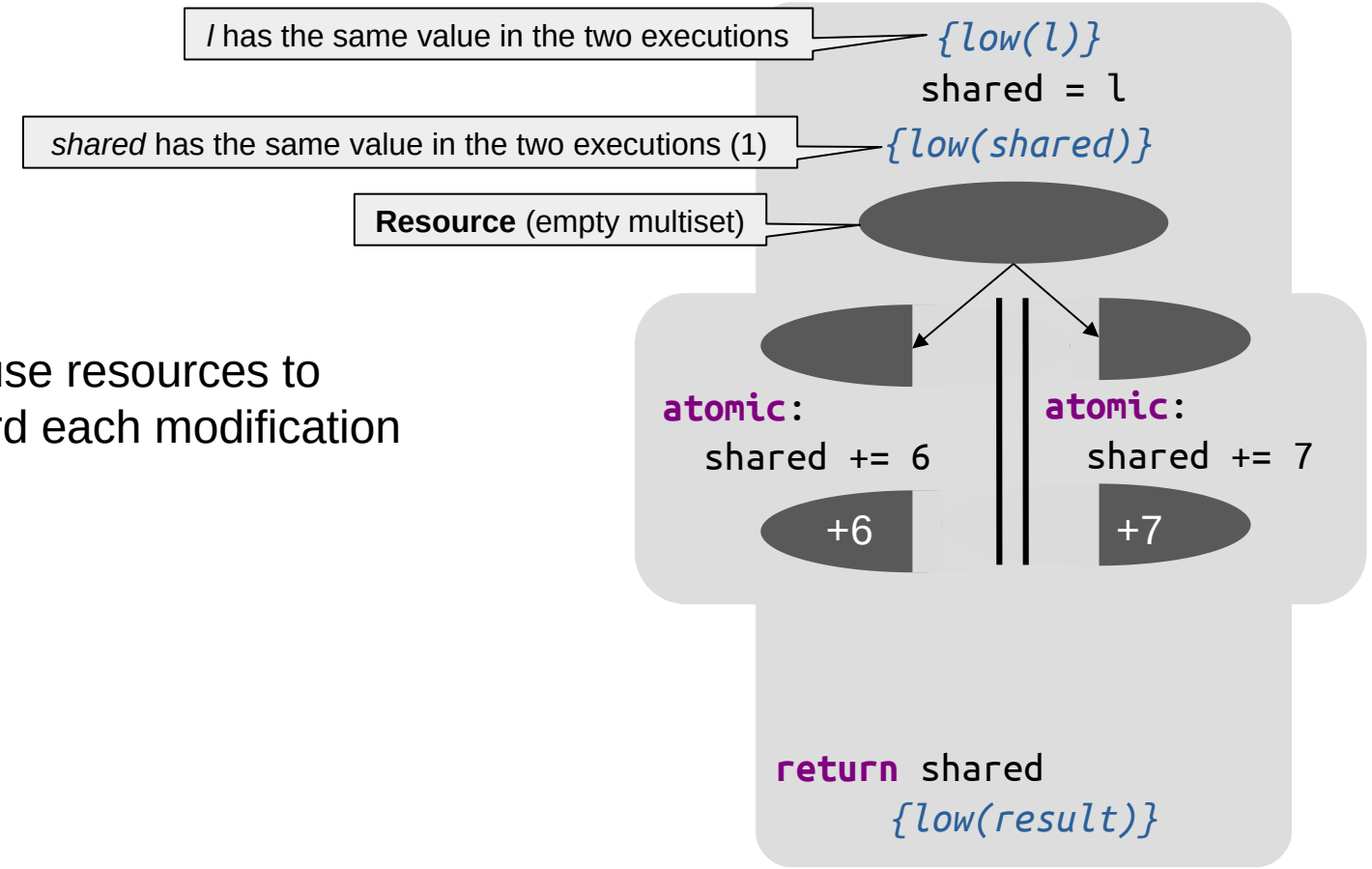

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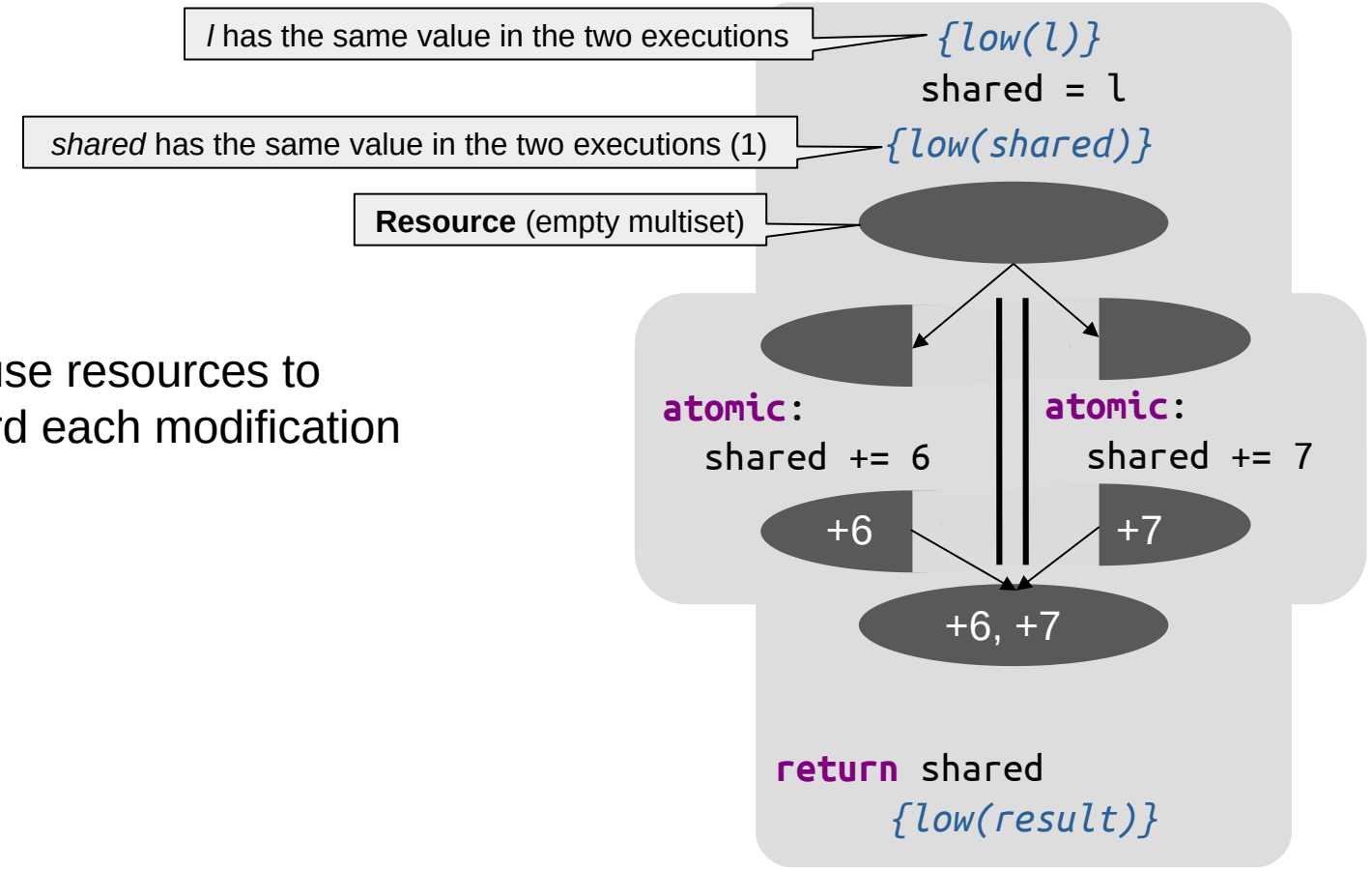


Verification Approach

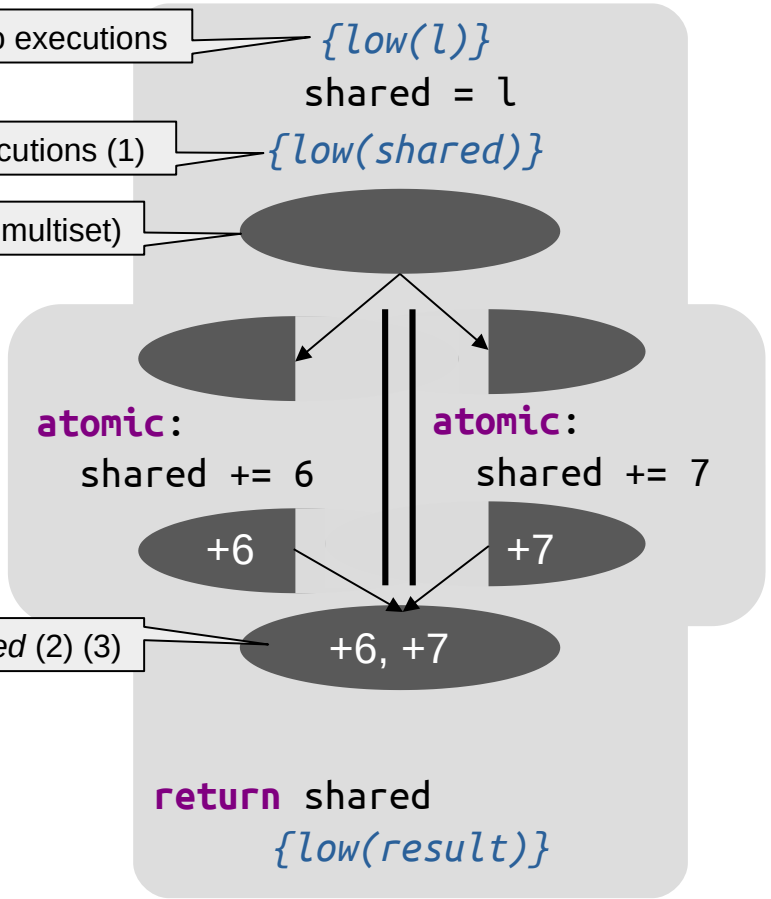
l has the same value in the two executions

shared has the same value in the two executions (1)

Resource (empty multiset)

We use resources to record each modification

Contains **all modifications** performed (atomically) on *shared* (2) (3)



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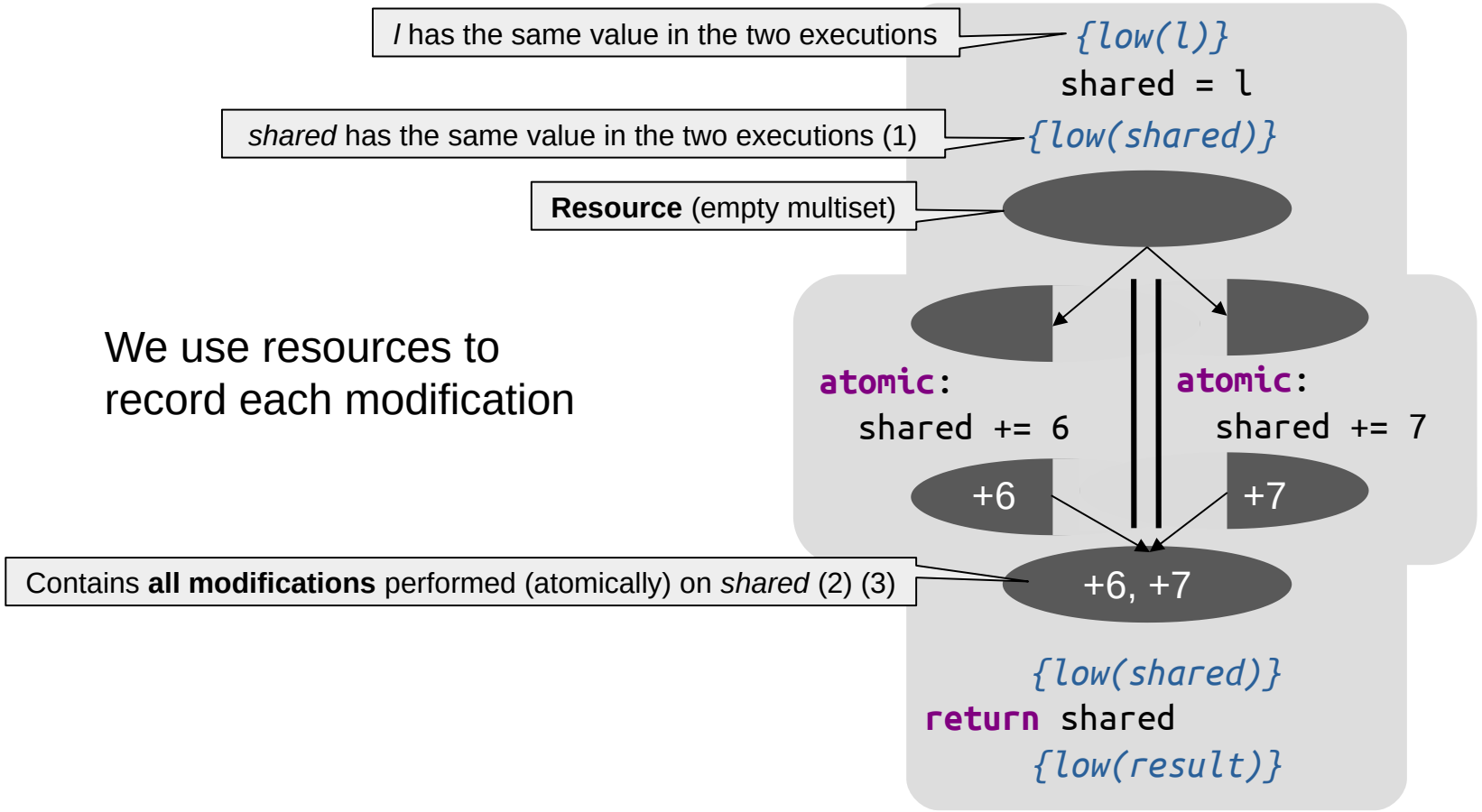

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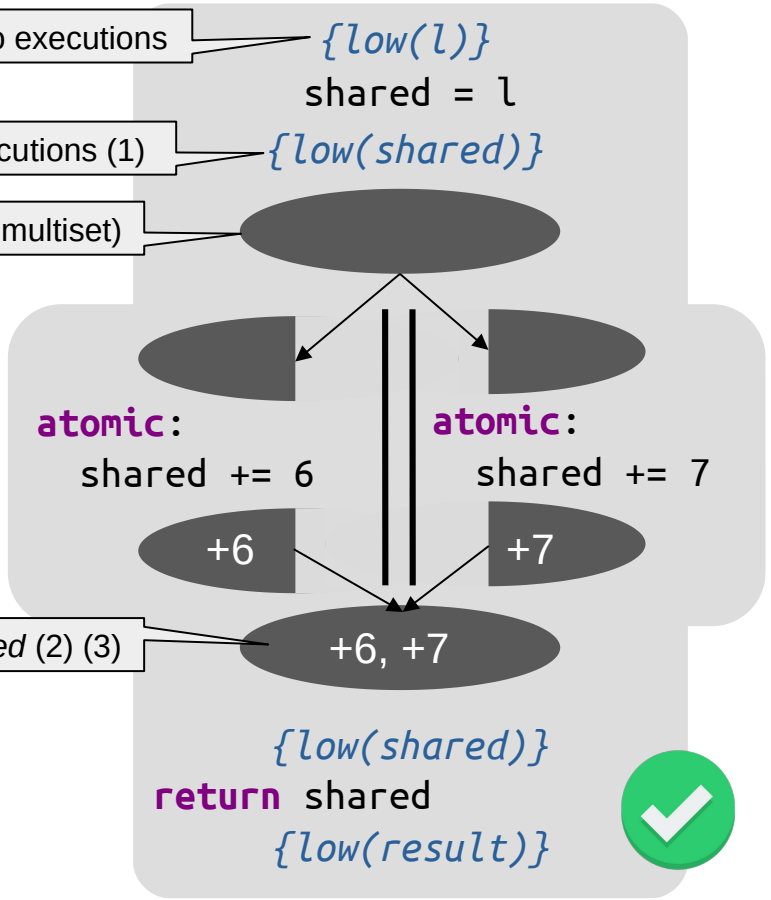

We can do better.

The Limits of Commutativity

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while i < h:
  i += 1
atomic:
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```

||

```
while j < 100:
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return sort(shared)
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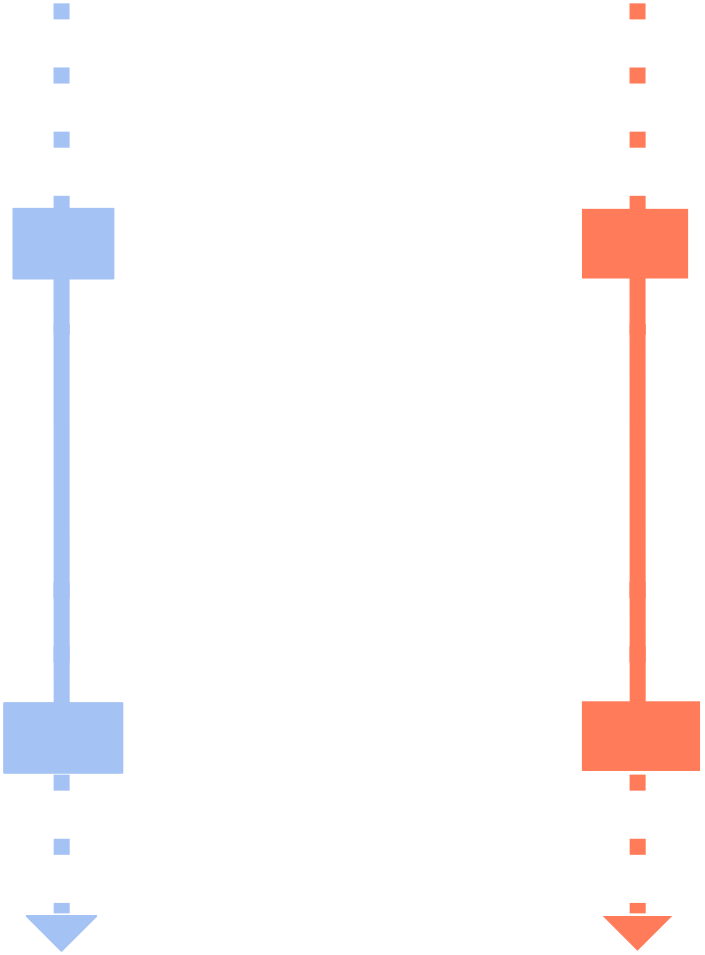

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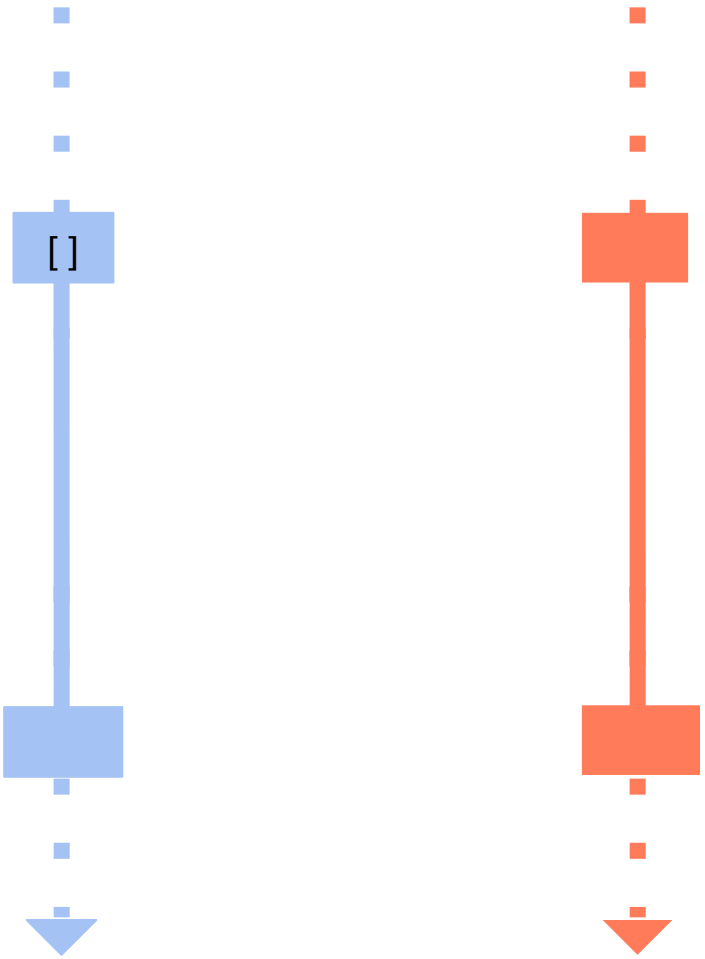

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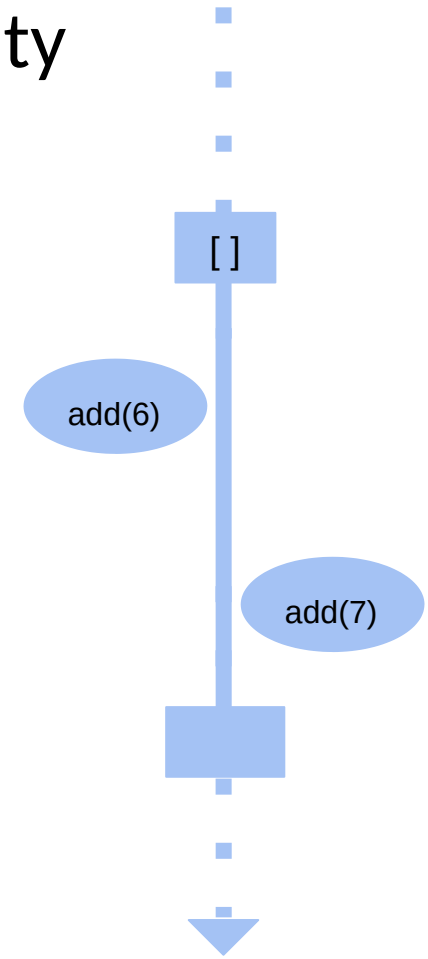

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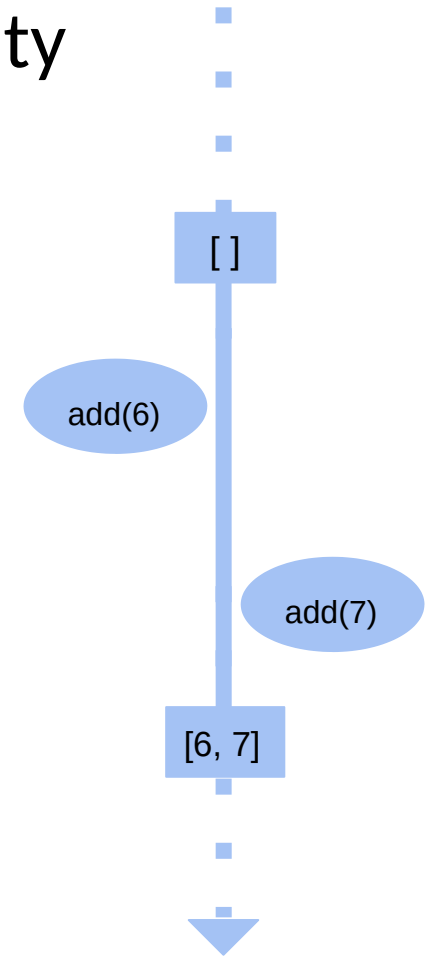

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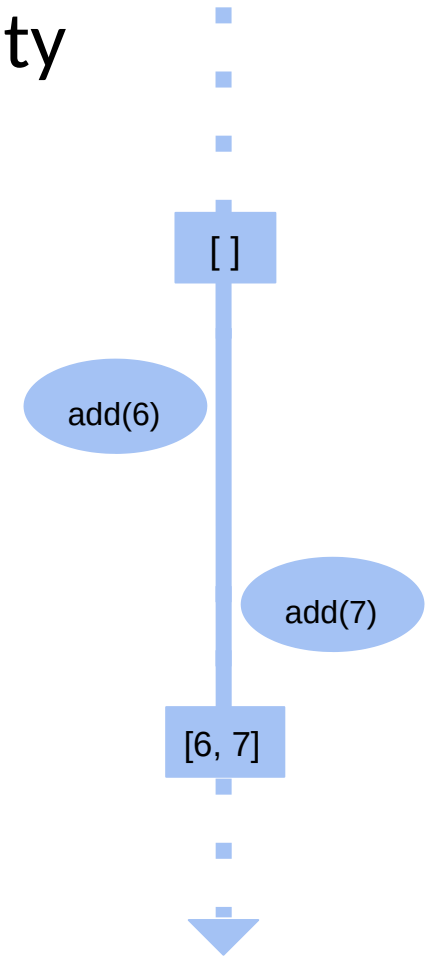

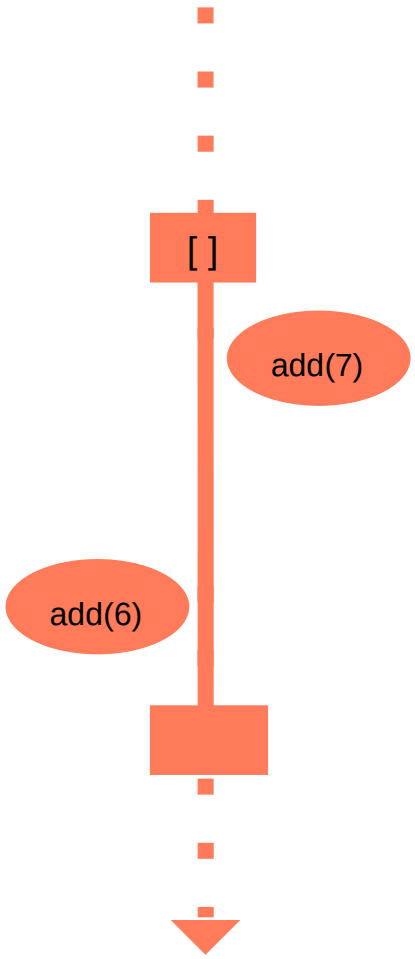

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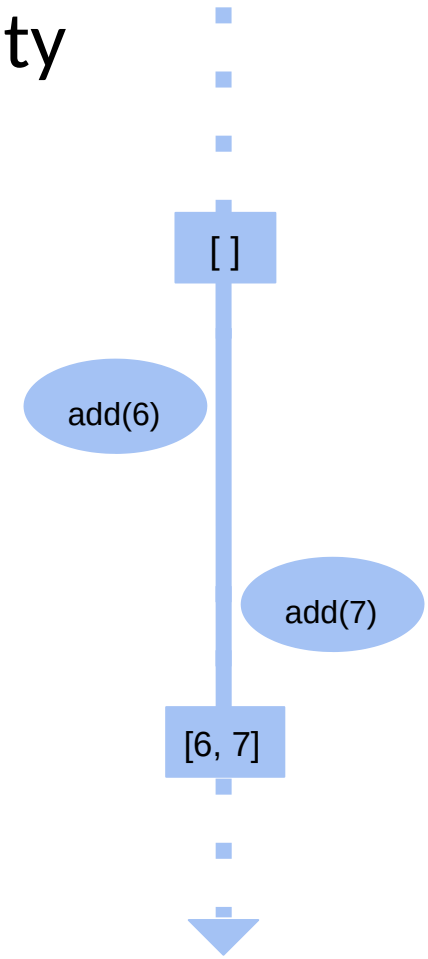

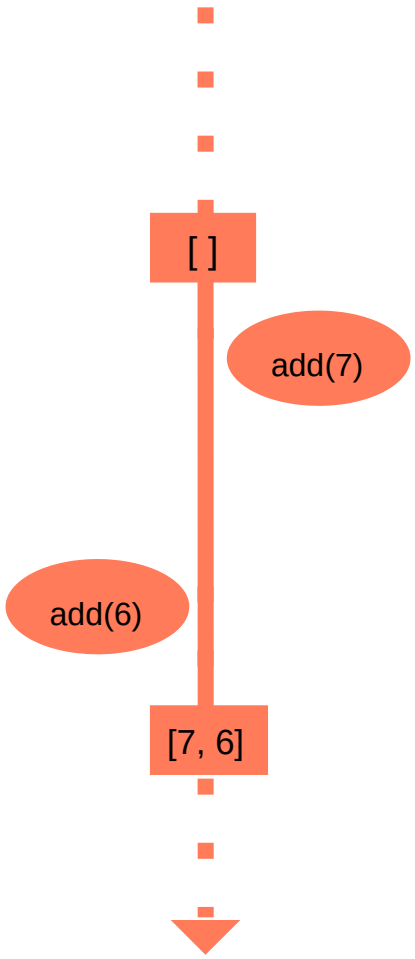

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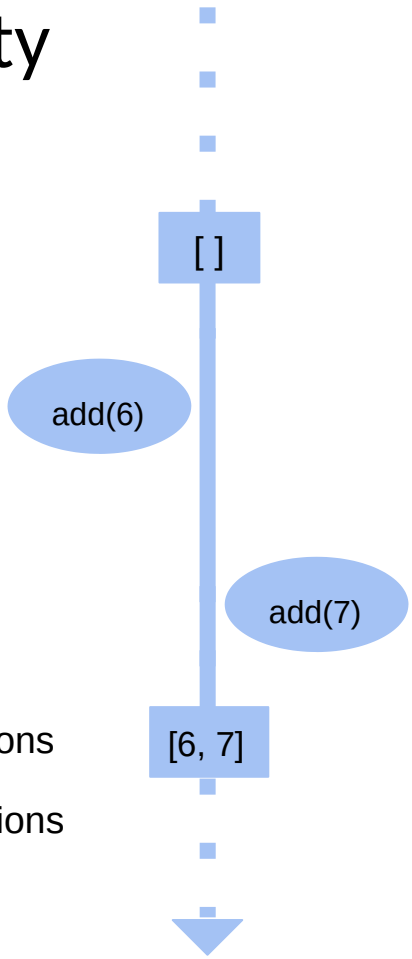

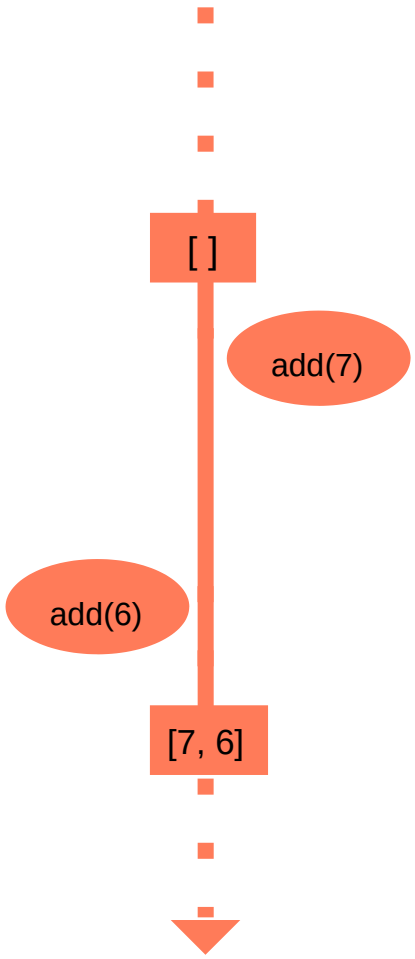

If

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(2) the two executions perform the “same” modifications

(3) the modifications commute

then *shared* has the same final value in both executions

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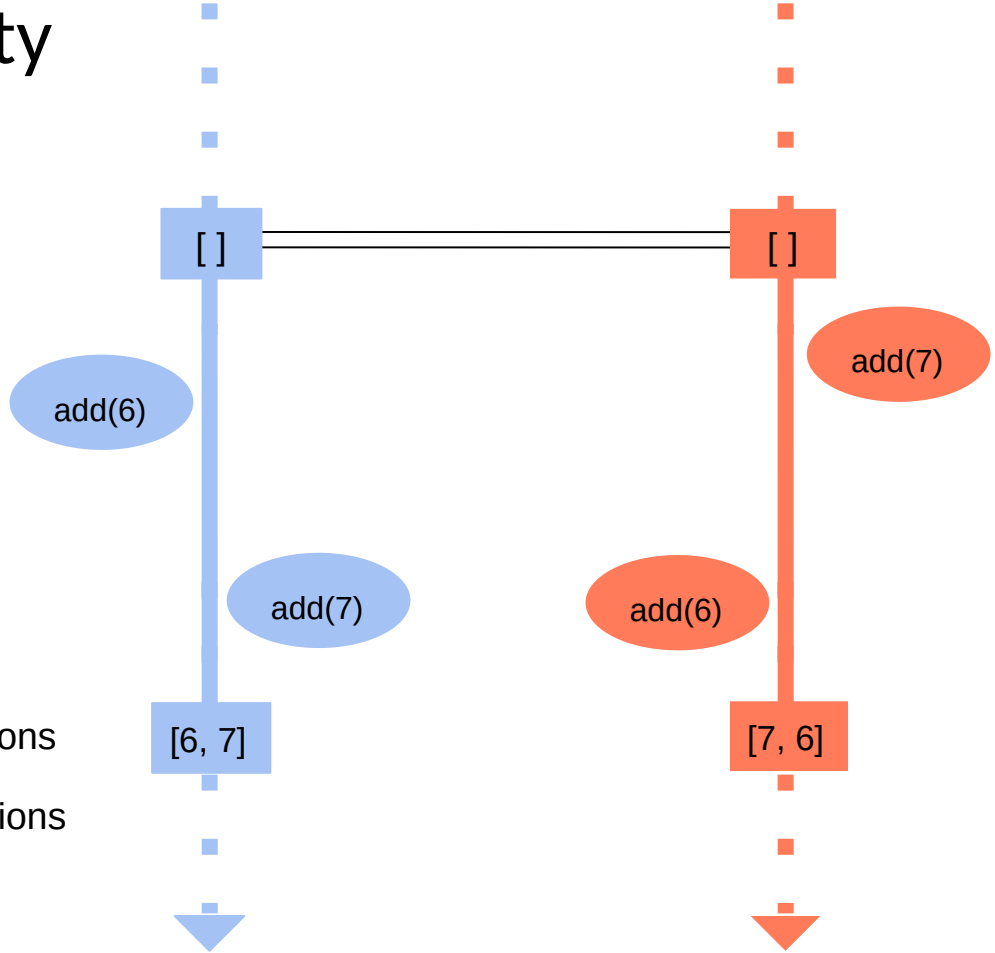

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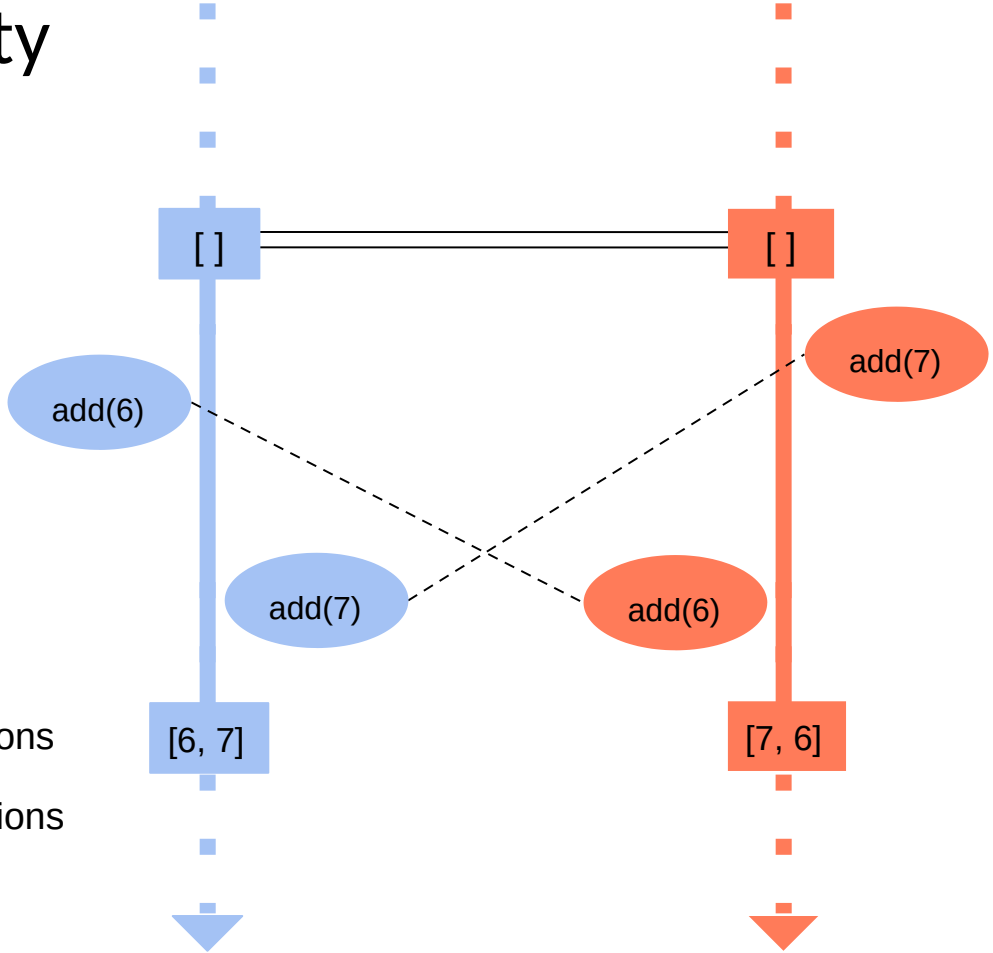

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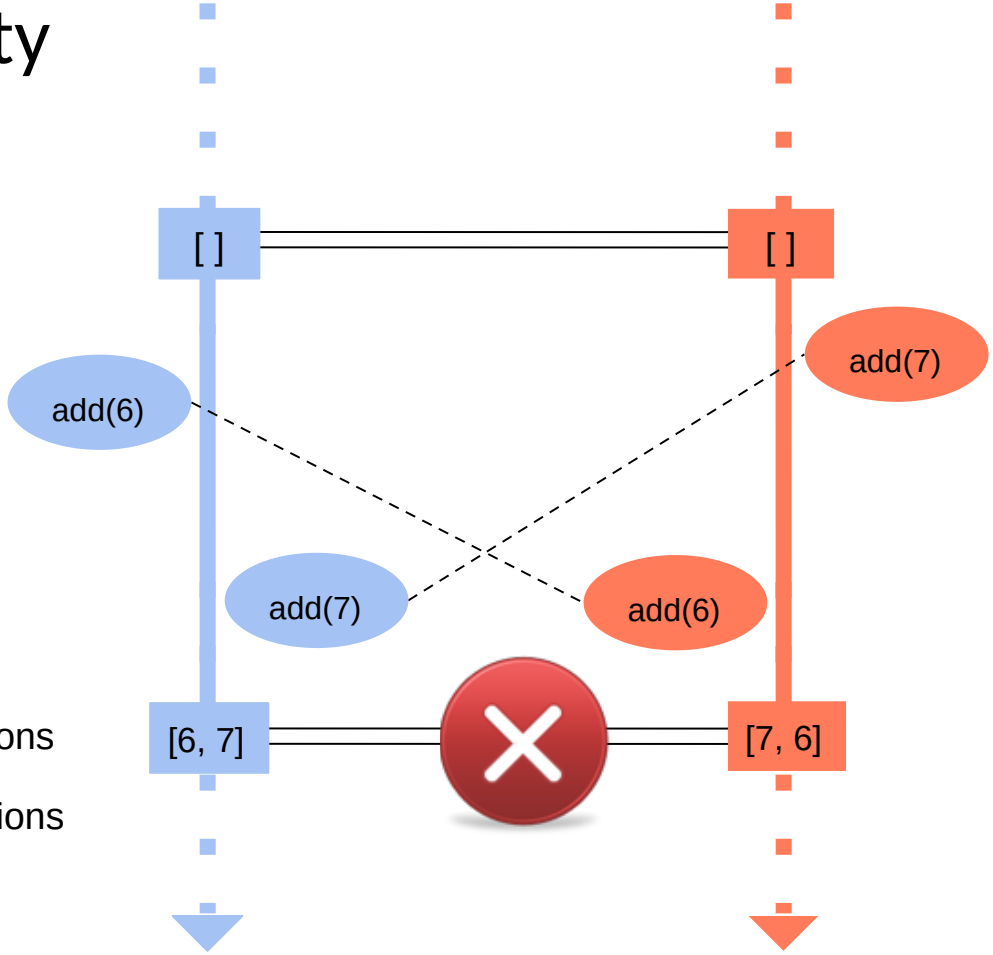

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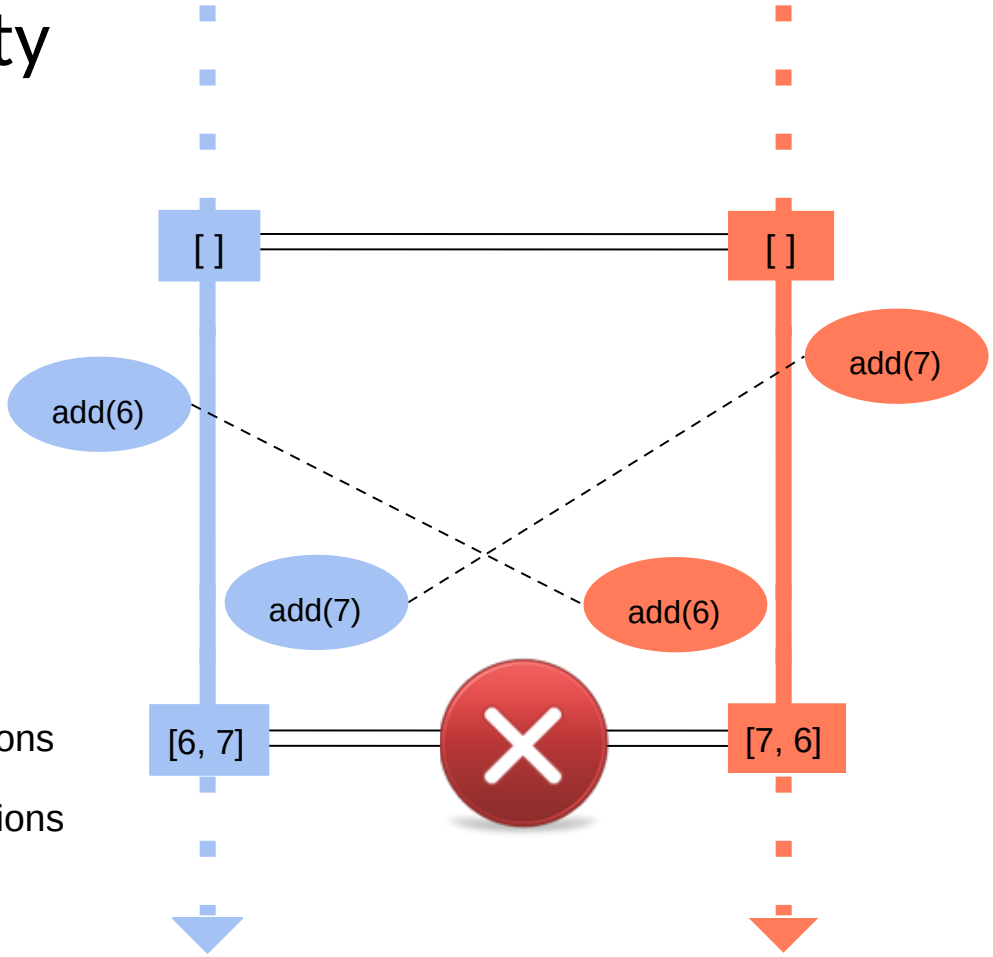

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Key Idea

Commutativity **modulo abstraction**

Commutativity Modulo Abstraction (“Abstract Commutativity”)

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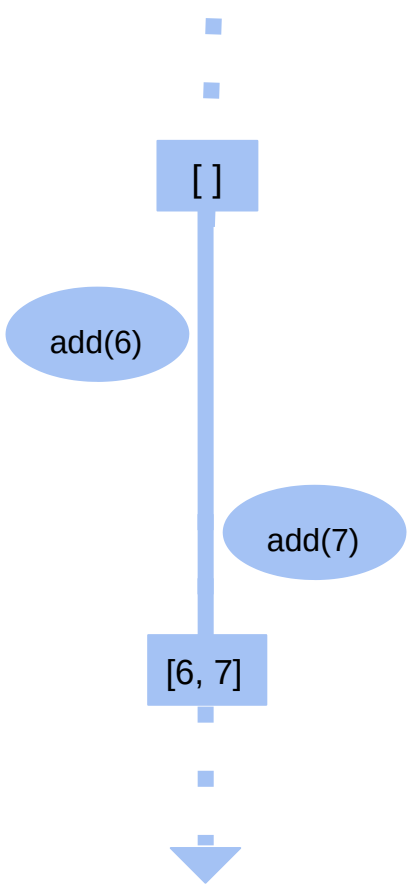

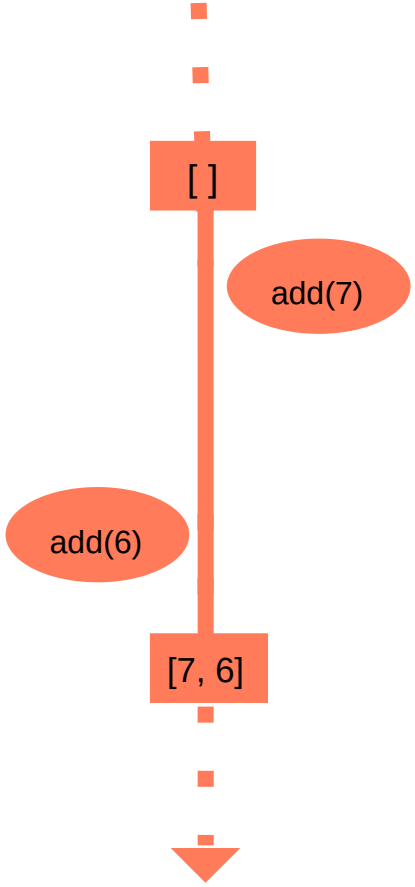

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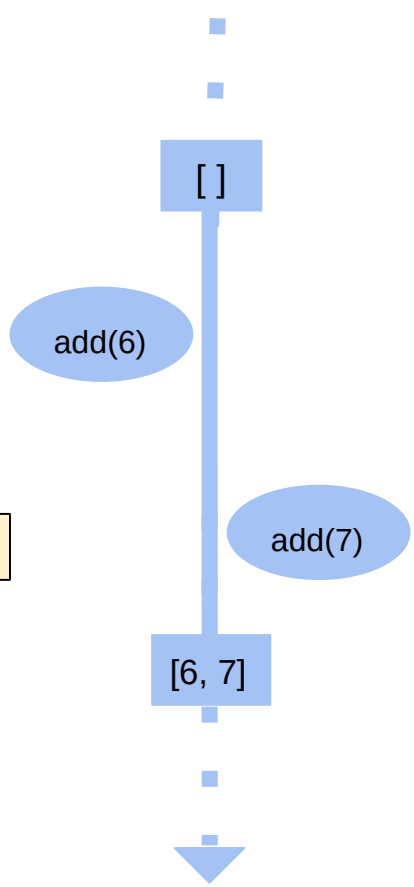
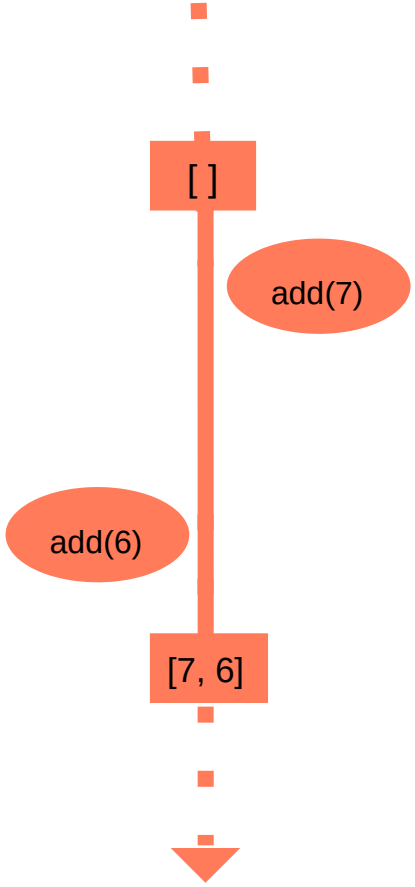
Abstraction **α**: list → multiset of elements





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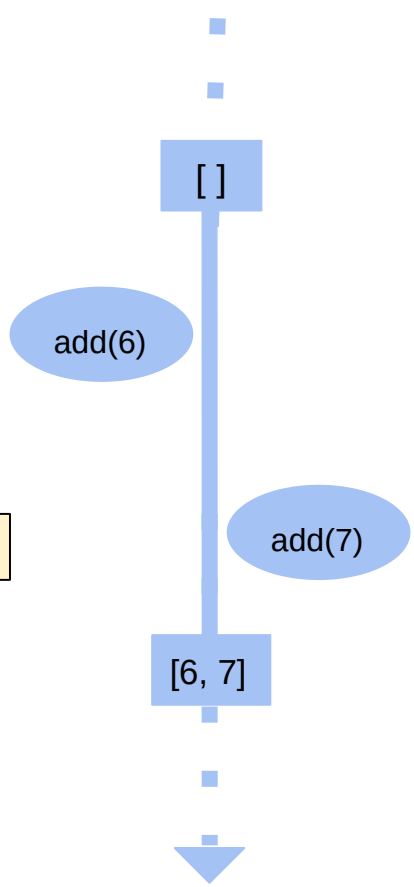

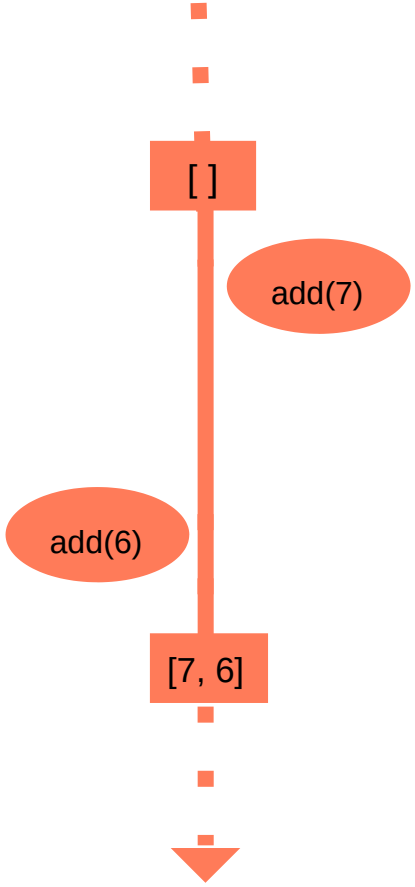

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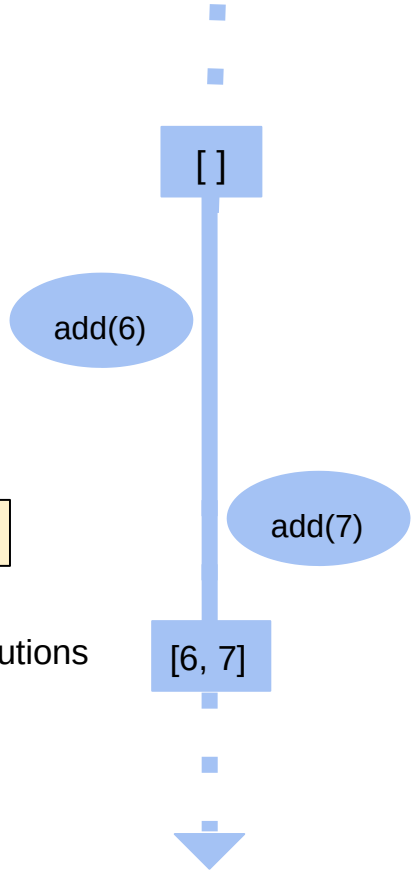

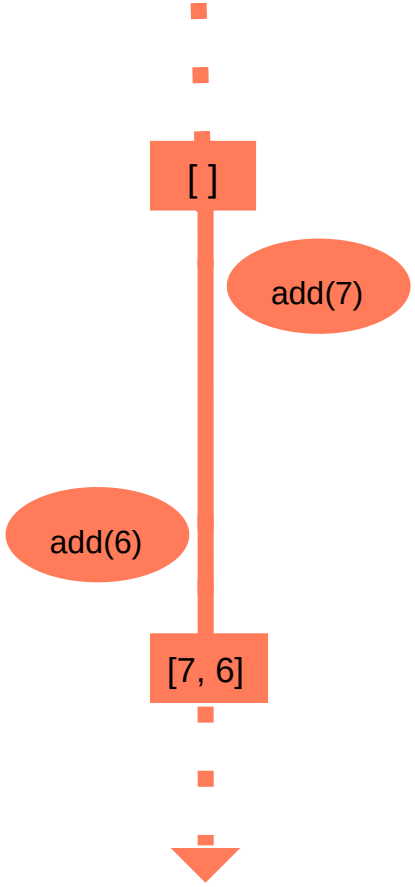

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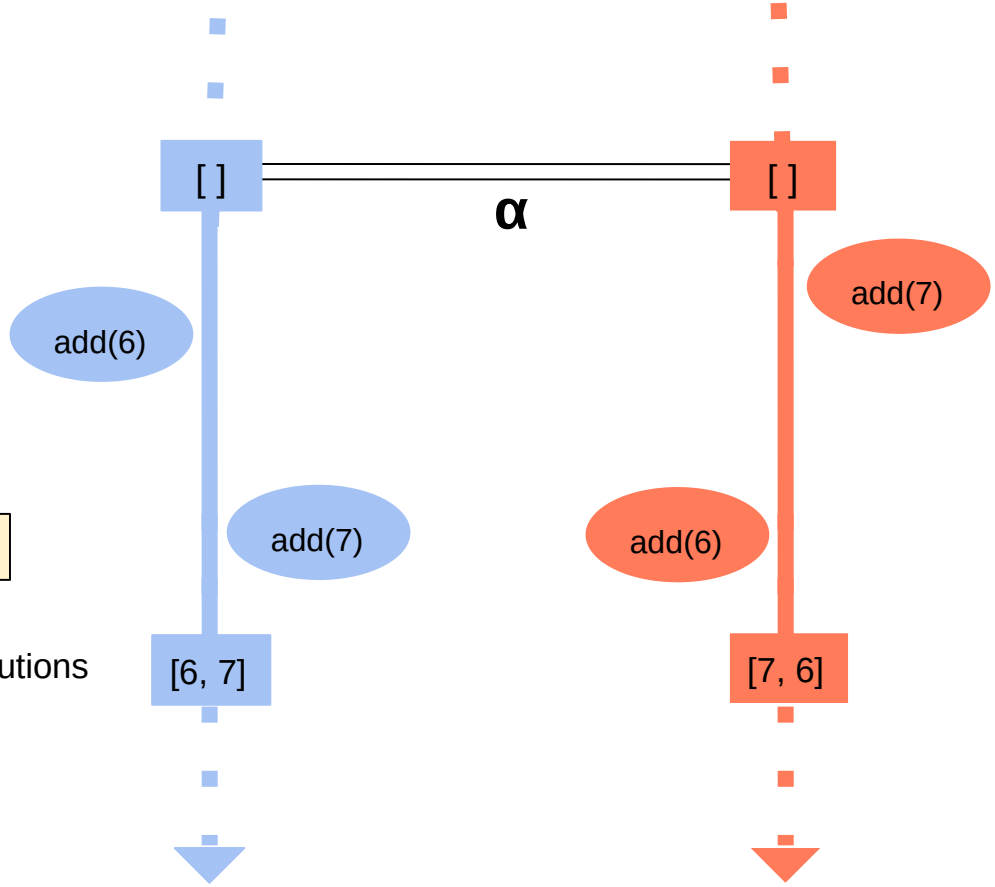

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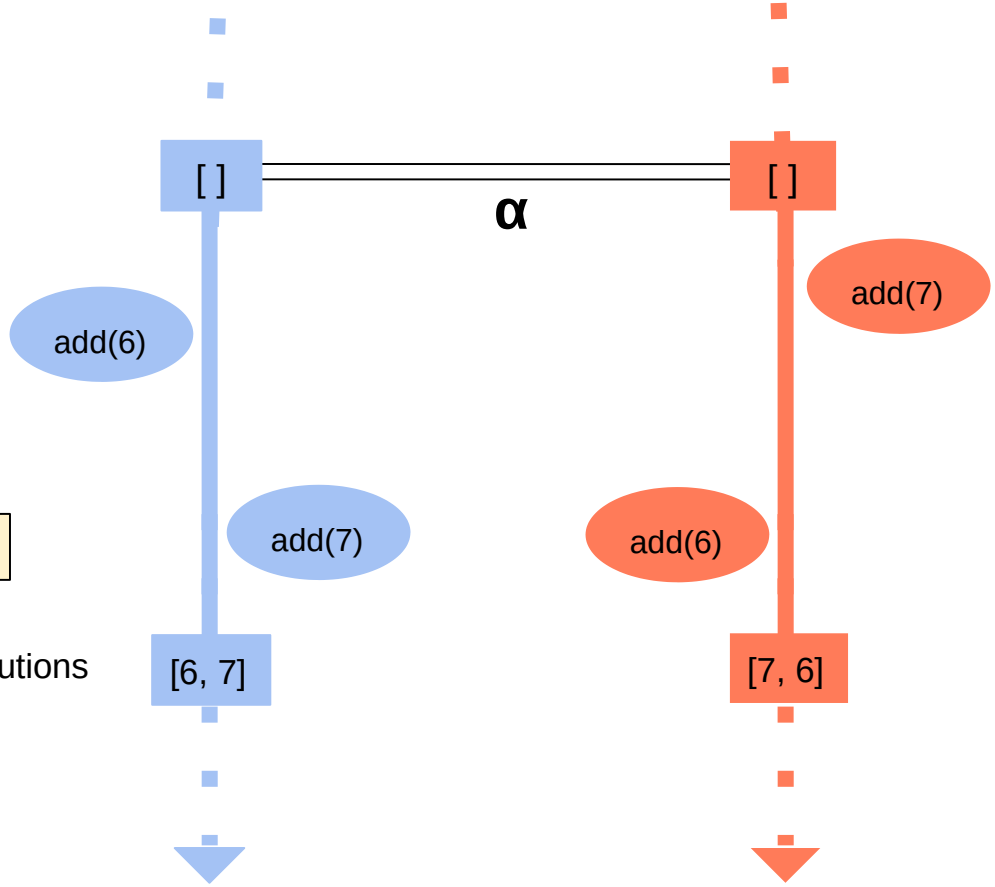

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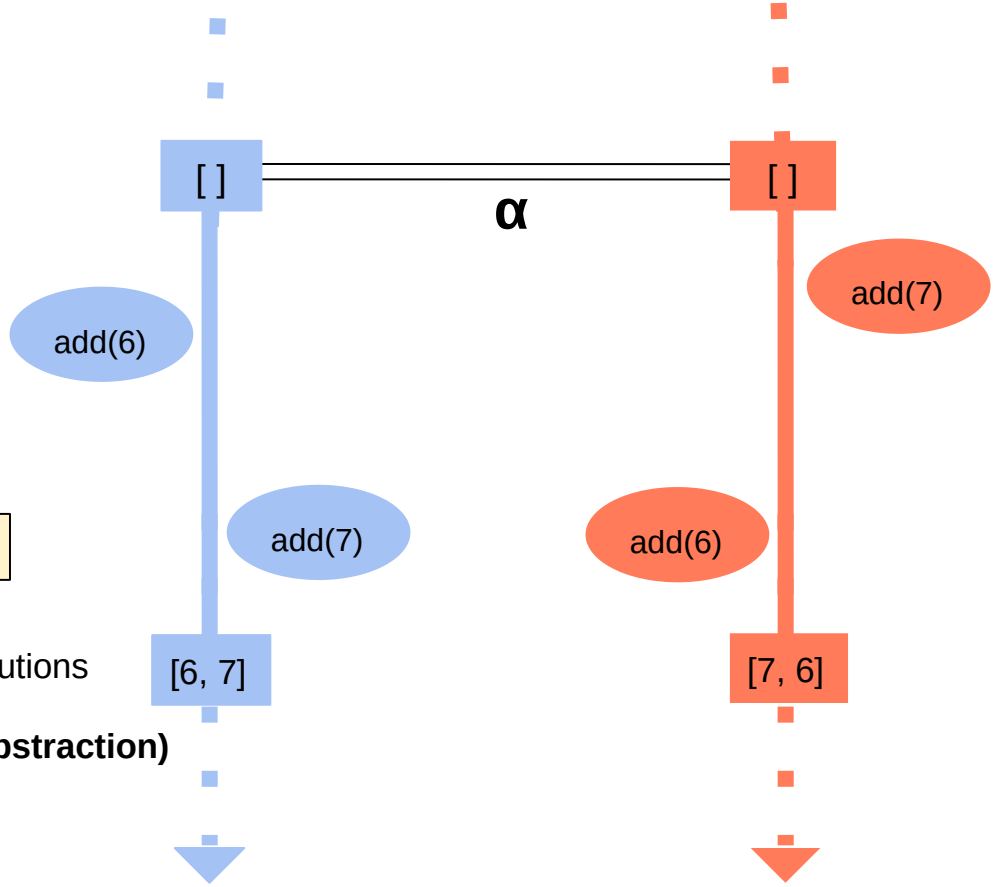

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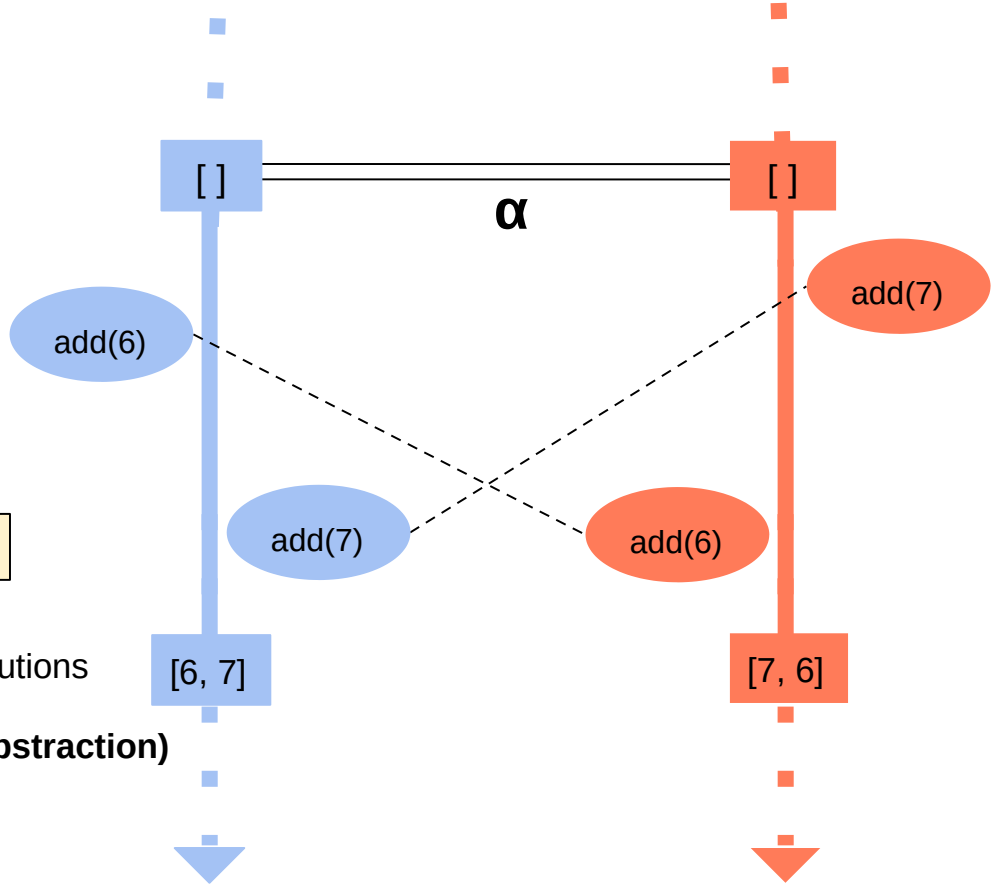

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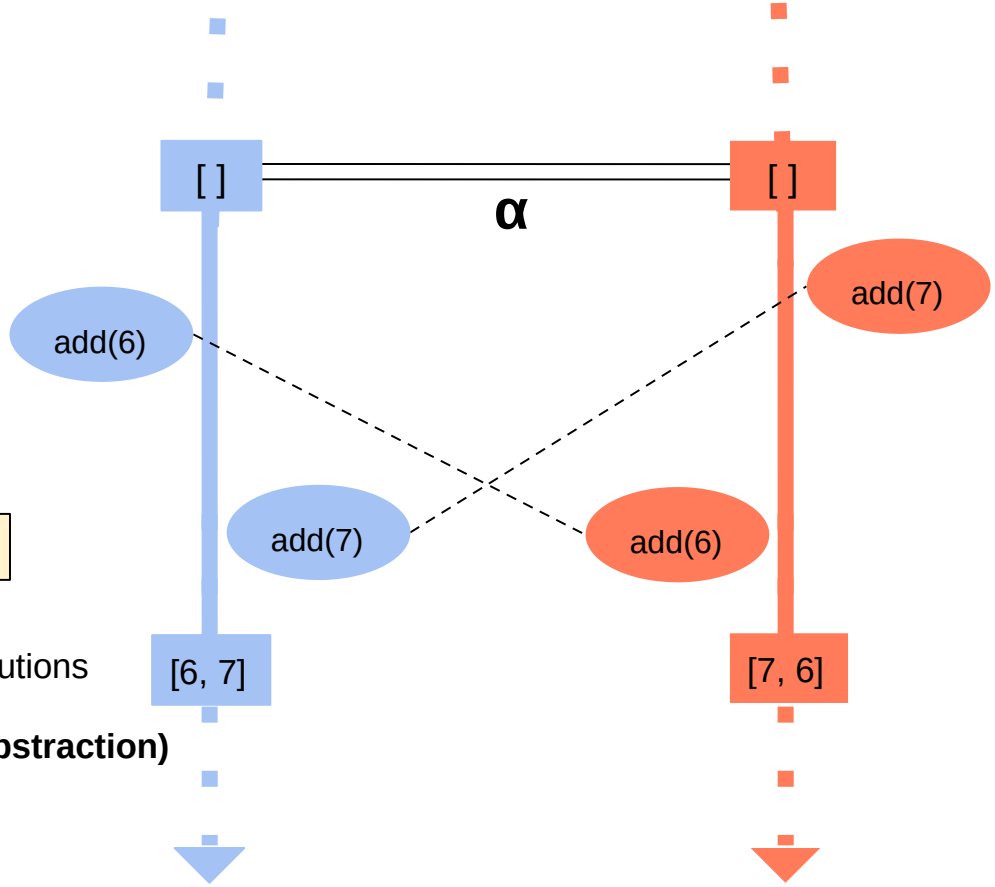

Abstraction **α**: list → multiset of elements

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- *shared* has the same initial **abstraction** in both executions
- executions perform “same” modifications (**modulo abstraction)**

(3) the modifications commute **modulo abstraction**

Commutativity Modulo Abstraction (“Abstract Commutativity”)

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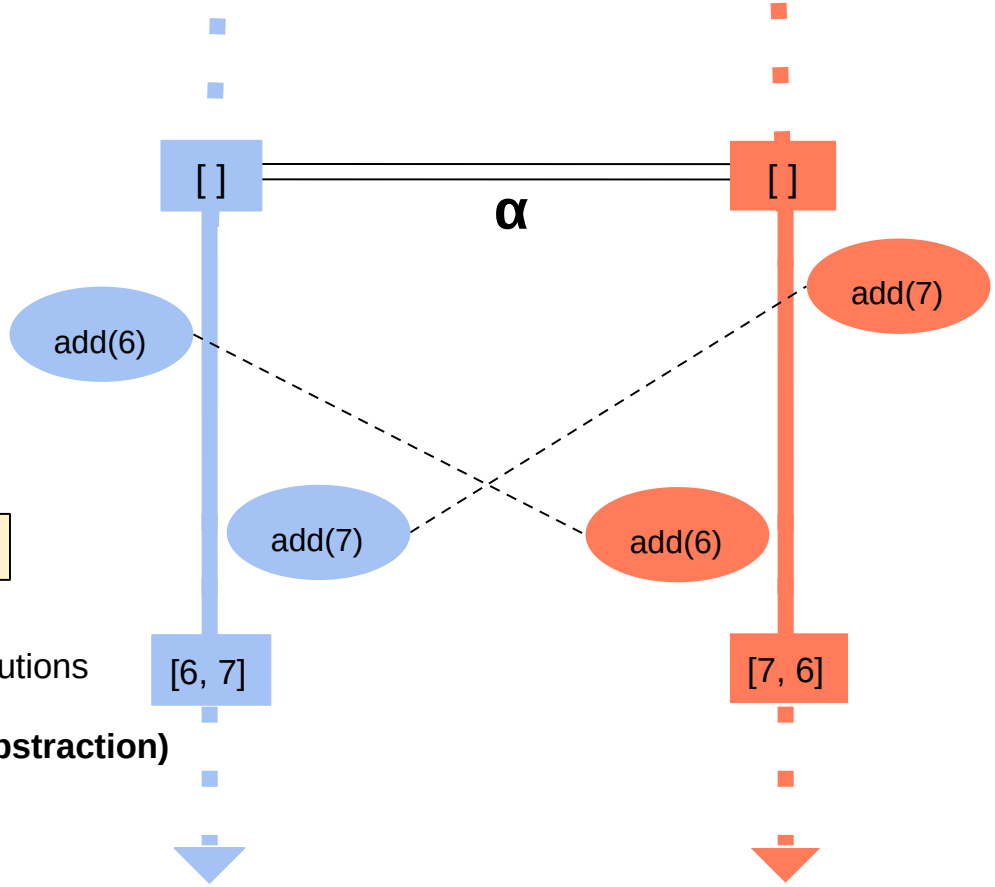

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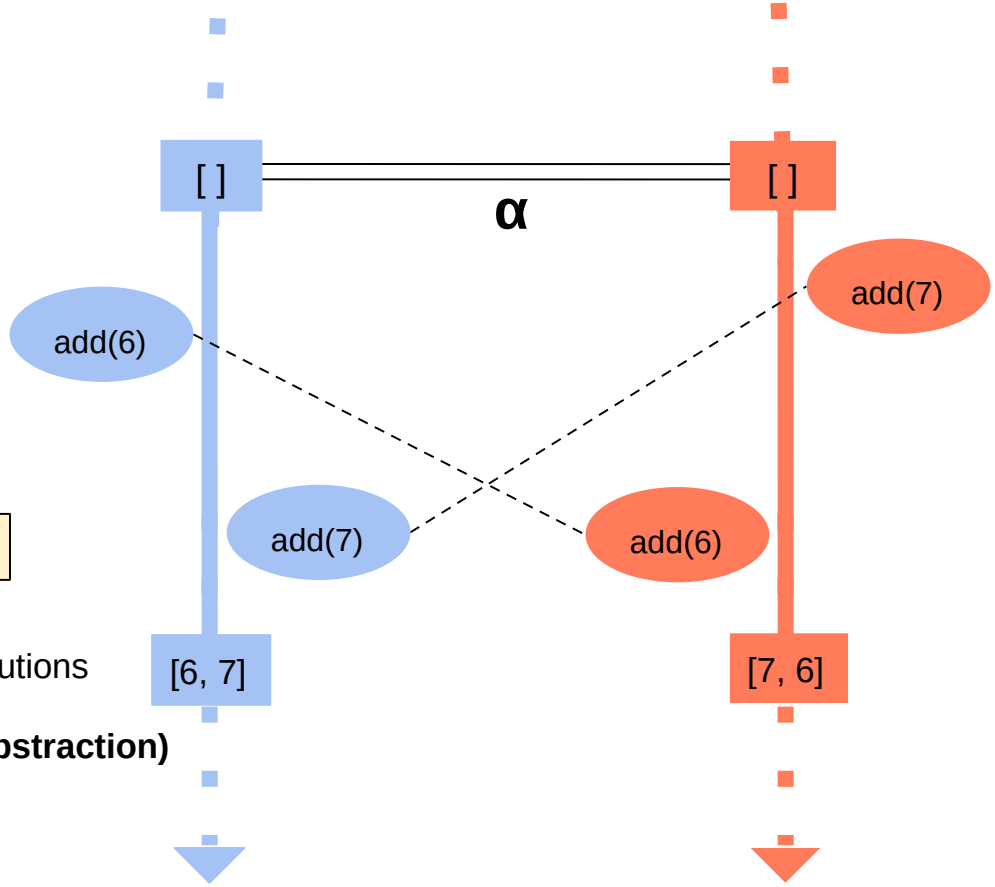

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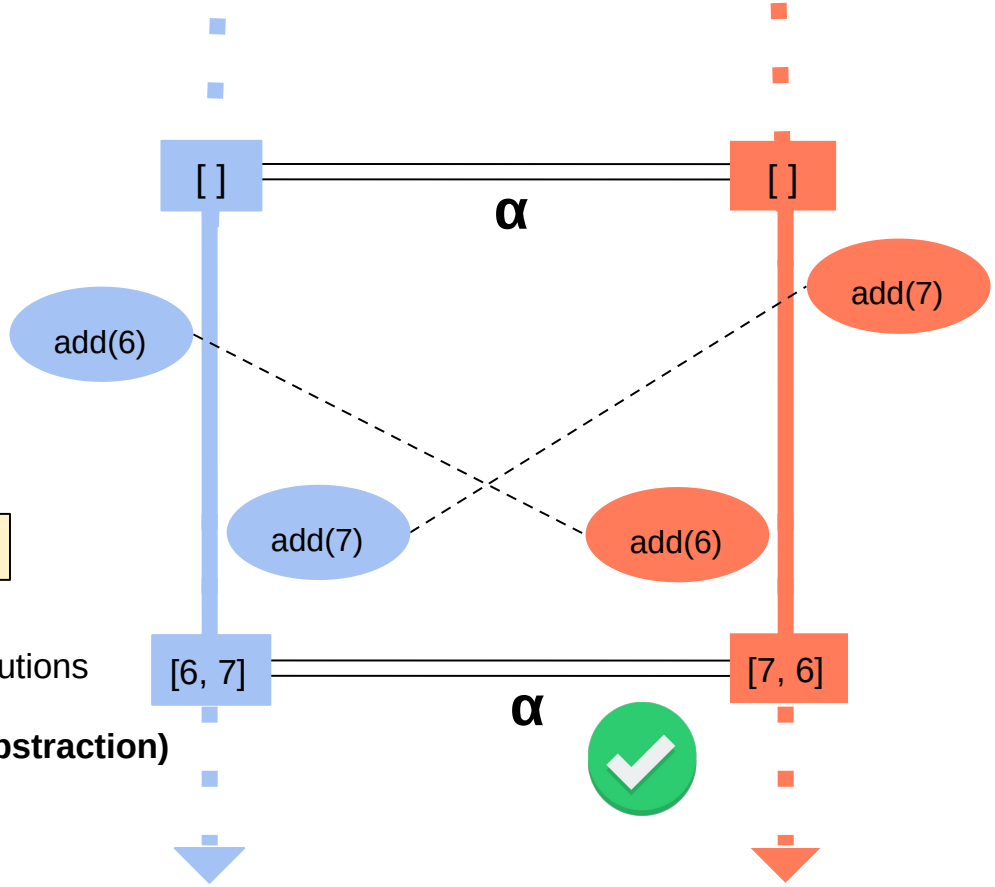

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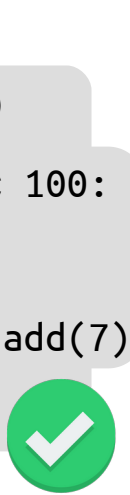

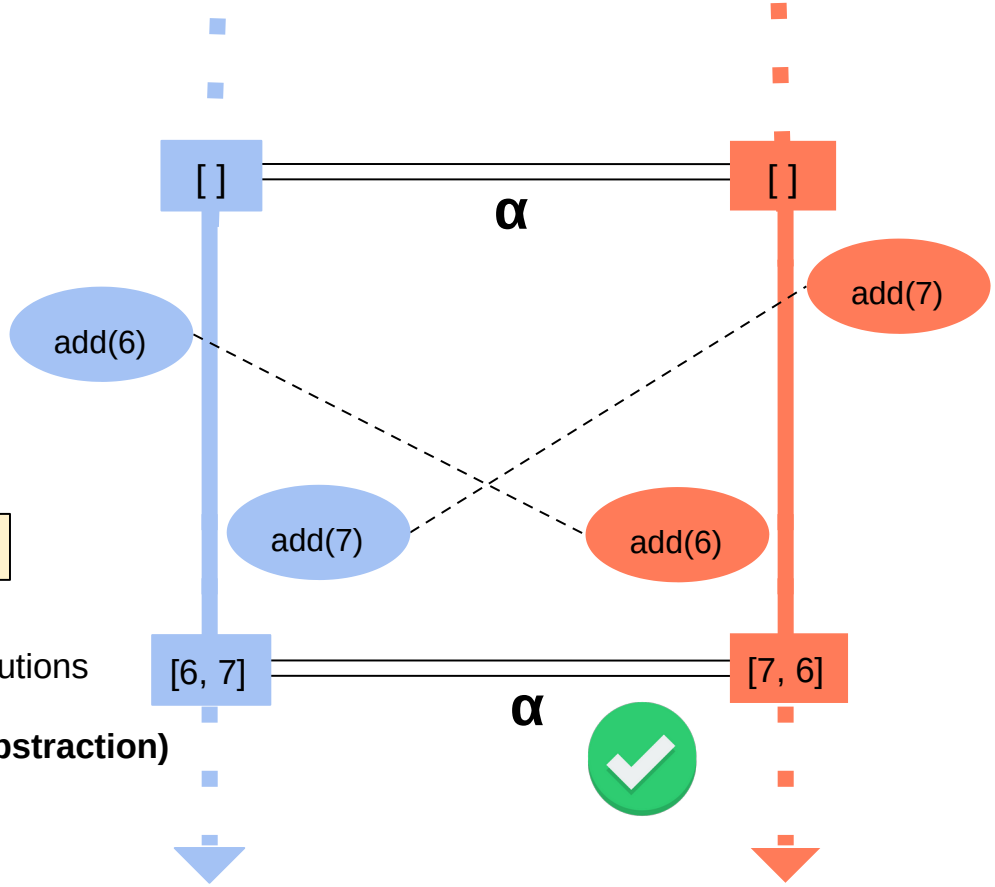

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return sort(shared)
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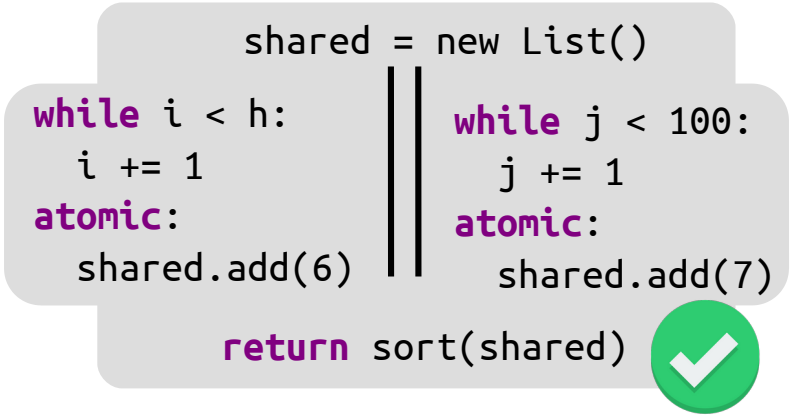

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shared = new Map()
while i < h:          ||  while j < 100:
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CommCSL

CommCSL

$$\Gamma \vdash \{P\} C \{Q\}$$

CommCSL

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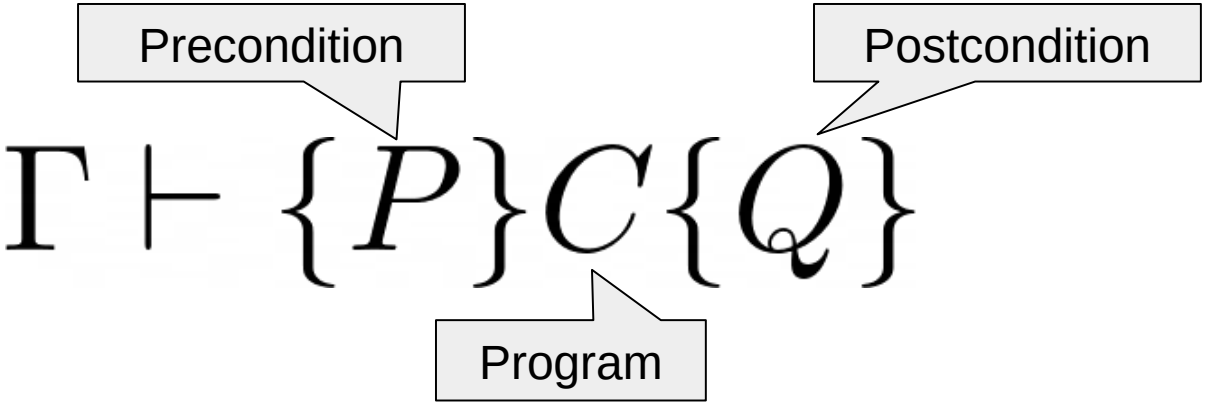
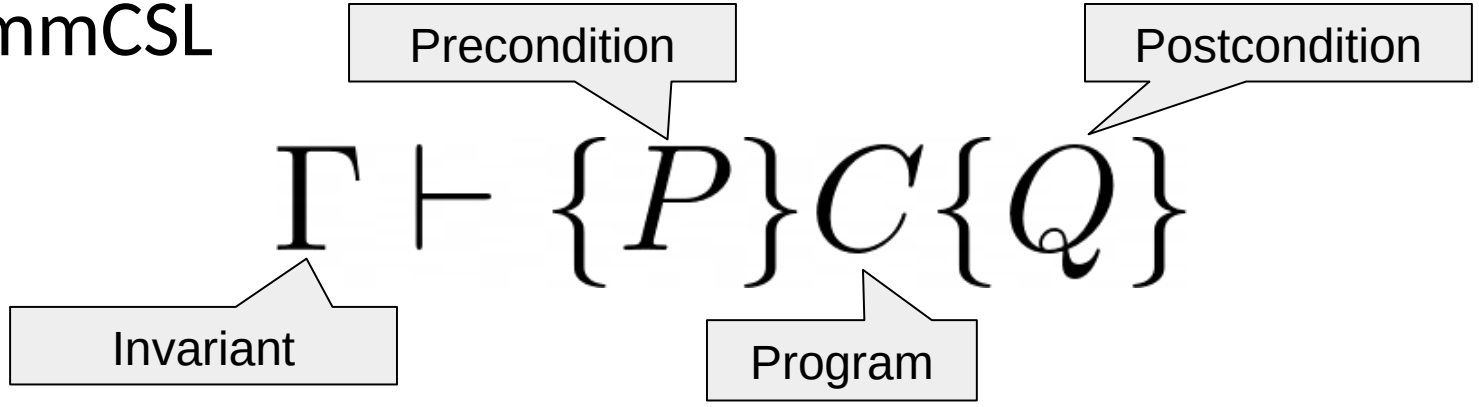
Program

CommCSL



CommCSL


Precondition
Postcondition
$\Gamma \vdash \{P\}C\{Q\}$
Invariant
Program

CommCSL

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$$\frac{\Gamma_\bot = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\bot \vdash \{P[e/x]\}x{:=}e\{P\}}\ (\textsc{Assign}) \qquad \frac{x \notin fv(e) \quad \Gamma_\bot = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\bot \vdash \{\mathtt{emp}\}x{:=}\mathtt{alloc}(e)\{x \mapsto^1 e\}}\ (\textsc{New})$$

$$\frac{x \notin fv(e_1, e_2) \quad \Gamma_\bot = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\bot \vdash \{e_1 \mapsto^r e_2\}x{:=}[e]\{e_1 \mapsto^r e_2 * x = e_2\}}\ (\textsc{Read}) \qquad \frac{}{\Gamma_\bot \vdash \{e_1 \mapsto^1 \_\}[e_1]{:=}e_2\{e_1 \mapsto^1 e_2\}}\ (\textsc{Write})$$

$$\frac{\Gamma_\bot \vdash \{P \wedge b\}c_1\{Q\} \quad \Gamma_\bot \vdash \{P \wedge \neg b\}c_2\{Q\}}{\Gamma_\bot \vdash \{P \wedge \mathrm{Low}(b)\}\mathtt{if}\ (b)\ \mathtt{then}\ \{c_1\}\ \mathtt{else}\ \{c_2\}\{Q\}}\ (\textsc{If1})$$

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$$\frac{\Gamma_\bot \vdash \{P_1\}c_1\{Q_1\} \quad \Gamma_\bot \vdash \{P_2\}c_2\{Q_2\} \quad fv(P_1, c_1, Q_1) \cap mod(c_2) = \emptyset \quad fv(P_2, c_2, Q_2) \cap mod(c_1) = \emptyset \quad \Gamma_\bot = \Gamma \Rightarrow fv(\Gamma) \cap mod(c_1, c_2) = \emptyset \quad P_1 \text{ is precise or } P_2 \text{ is precise}}{\Gamma_\bot \vdash \{P_1 * P_2\}c_1 \parallel c_2\{Q_1 * Q_2\}}\ (\textsc{Par})$$

$$\frac{\Gamma_\bot \vdash \{P\}c\{Q'\} \quad Q' \Rightarrow Q}{\Gamma_\bot \vdash \{P\}c\{Q\}}\ (\textsc{Cons}) \qquad \frac{fv(K) \cap mod(c) = \emptyset \quad \Gamma_\bot \vdash \{P\}c\{Q\} \quad P \text{ is precise or } R \text{ is precise}}{\Gamma_\bot \vdash \{P * R\}c\{Q * R\}}\ (\textsc{Frame})$$

$$\frac{x \notin fv(c) \quad \Gamma_\bot = \Gamma \Rightarrow x \notin fv(\Gamma) \quad \Gamma_\bot \vdash \{P\}c\{Q\}}{\Gamma_\bot \vdash \{\exists x.\, P\}c\{\exists x.\, Q\}}\ (\textsc{Exists})$$

$$\frac{\Gamma = \langle \alpha, f_{a_s}, f_{a_u}, I(x)\rangle \quad \Gamma \text{ is valid} \quad I(x) \text{ is unary and precise} \quad \Gamma \vdash \{P * \mathit{sguard}(1, \emptyset^\#) * \mathit{uguard}([\,])\}c\{Q * \mathit{sguard}(1, x_s) * \mathit{PRE}_s(x_s) * \mathit{uguard}(x_u) * \mathit{PRE}_u(x_u)\}}{\bot \vdash \{I(x) * \mathrm{Low}(\alpha(x)) * P\}c\{\exists x'.\, I(x') * \mathrm{Low}(\alpha(x')) * Q\}}\ (\textsc{Share})$$

$$\frac{\Gamma = \langle \alpha, f_{a_s}, f_{a_u}, I(x)\rangle \quad I(x_v) \text{ is unary and precise} \quad x_v \notin fv(P, Q) \quad x_s, x_a, x_v \notin mod(c) \quad \mathit{noguard}(P) \quad \mathit{noguard}(Q) \quad \bot \vdash \{P * I(x_v)\}c\{Q * I(f_{a_s}(x_v, x_a))\}}{\Gamma \vdash \{P * \mathit{sguard}(r, x_s)\}\mathtt{atomic}\ c\{Q * \mathit{sguard}(r, x_s \cup^\# \{x_a\}^\#)\}}\ (\textsc{AtomicShr})$$

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CommCSL

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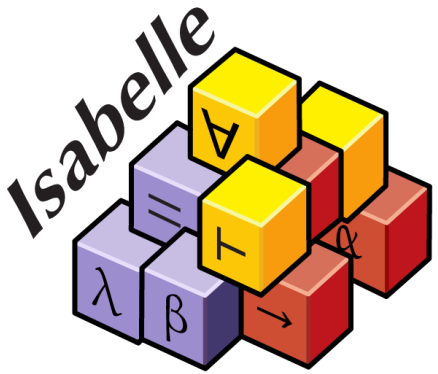
- Relational concurrent separation logic
- Support for (abstract) commutativity-based information flow reasoning
- Thread-modular reasoning, mutable heaps

- Other features:
  - Low events, standard output…
  - More complete support for non-symmetric concurrency

- Formalized and proved sound in Isabelle/HOL
  - Challenging soundness argument distinct from existing logics
  - Available on the Archive of Formal Proofs



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Non-interference theorem

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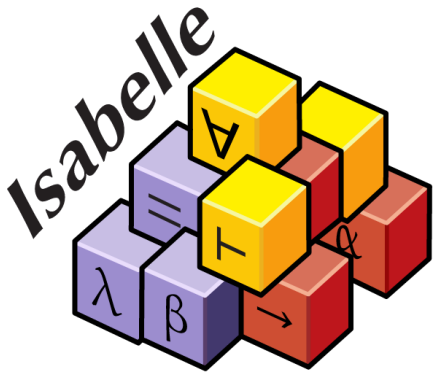

Implementation

HyperViper

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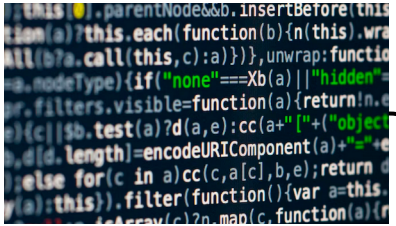
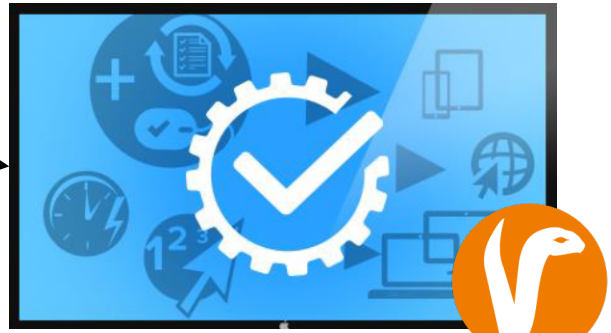


Source code



HyperViper

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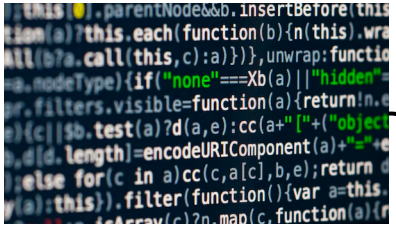
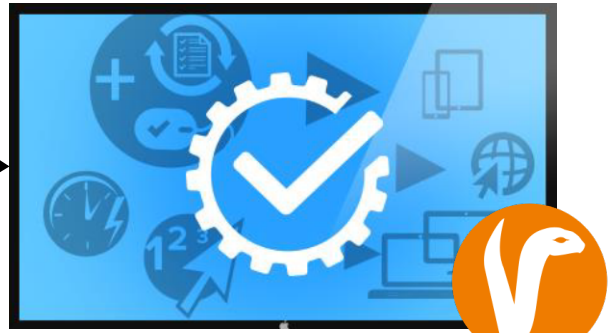


Source code

Specification
(e.g., low variables and data)



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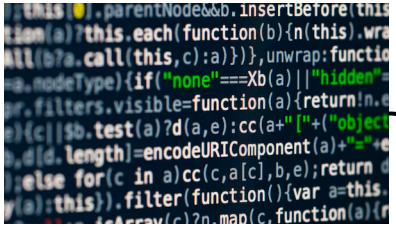

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Hints (e.g., abstractions)

HyperViper

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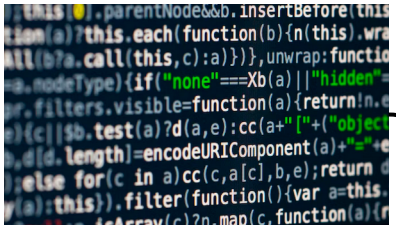

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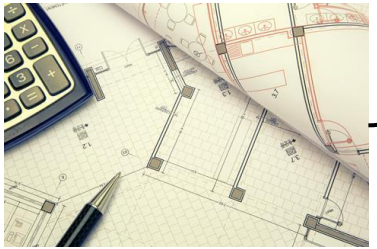

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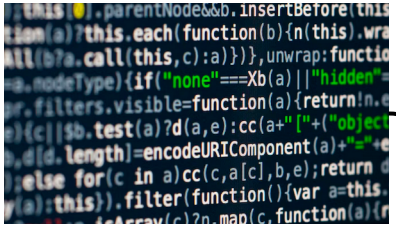
Hints (e.g., abstractions)

HyperViper

No information leak through values
(in all executions)

Implementation



Source code

Specification
(e.g., low variables and data)

Hints (e.g., abstractions)

HyperViper

No information leak through values
(in all executions)

Program **might** leak secret data
(in at least one execution)

HyperViper

- Automated, SMT-based verifier
  - Based on Viper verification infrastructure and Z3 
  - Relational reasoning using Modular Product Programs
- User provides abstractions, pre- and postconditions, invariants…
- Supports dynamic thread creation, multiple shared resources, …
- https://github.com/viperproject/hyperviper

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```
lockType IntLock {
 type Int
 invariant(l, v) = [l.lockInt |-> ?cp && [cp.val |-> v]]
 alpha(v): Int = 0  // we abstract to a constant, so everything commutes
 actions = [(SetValue, Int, duplicable)]
 action SetValue(v, arg)
   requires true
 { arg }
 noLabels = 2
}

...

method worker(l: Lock, lbl: Int)
 requires lowEvent && sguard[IntLock,SetValue](l, Set(lbl))
 requires sguardArgs[IntLock,SetValue](l, Set(lbl)) == Multiset[Int]()
 ensures sguard[IntLock,SetValue](l, Set(lbl))
 ensures allPre[IntLock, SetValue](sguardArgs[IntLock,SetValue](l, Set(lbl)))
{

   var v: Int
   v := lbl
   with[IntLock] l performing SetValue(v) at lbl {
       l.lockInt.val := v
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}

method print(i: Int)
 requires lowEvent && low(i)
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Evaluation

Example	Data structure	Abstraction	LOC	Ann.	T
Count-Vaccinated	Counter, increment	None	44	46	10.15
Figure 2	Integer, add	None	129	95	10.90
Count-Sick-Days	Integer, add	None	52	45	13.67
Figure 1	Integer, arbitrary	Constant	29	20	1.52
Mean-Salary	List, append	Mean	80	84	14.10
Email-Metadata	List, append	Multiset	82	75	16.70
Patient-Statistic	List, append	Length	73	70	4.92
Debt-Sum	List, append	Sum	76	81	14.45
Sick-Employee-Names	Treeset, add	None	105	113	28.43
Website-Visitor-IPs	Listset, add	None	74	69	6.20
Figure 3	HashMap, put	Key set	129	96	10.37
Sales-By-Region	HashMap, disjoint put	None	129	104	12.37
Salary-Histogram	HashMap, increment value	None	135	109	13.78
Count-Purchases	HashMap, add value	None	137	109	11.73
Most-Valuable-Purchase	HashMap, conditional put	None	140	118	17.87
1-Producer-1-Consumer	Queue	Consumed sequence	82	88	3.23
Pipeline	Two queues	Consumed sequences	122	100	3.66
2-Producers-2-Consumers	Queue	Produced multiset	130	134	8.45

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Secret data influences which thread performs which modification

- Modular reasoning about value sensitivity for concurrent programs
  - Independently of timing
  - Sound on real hardware
- Key idea is to exploit commutativity modulo abstraction
- Proved sound in Isabelle/HOL, automated in prototype verifier
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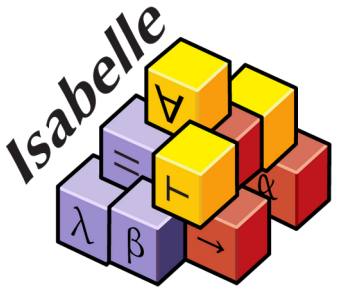

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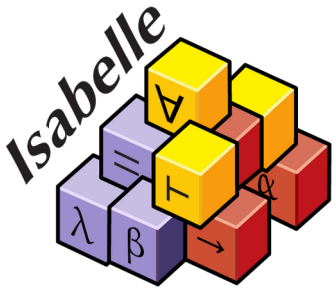

Thank you for your attention!

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